Abstract

Part 1:
How do closed quantum many-body systems driven out of equilibrium eventually achieve equilibration? In the first part of this thesis, we focus on the problem of relaxation of quantum state and typicality of quantum measurements, and its connected question *how quantum states relax and reach equilibrium?*. We do it by analyzing timescales of both equilibration and decoherence, which are one of the most fundamental questions in the field of quantum thermodynamics.

We analyze equilibration times of subsystems of a larger system under a random total Hamiltonian, in which the basis of the Hamiltonian is drawn from the Haar measure. It is known that in this situation, most of states of the subsystem will equilibrate, but not a lot is known about timescales of such a process. We obtain that the time of equilibration is of the order of the inverse of the arithmetic average of the Bohr frequencies, thus showing that typical times of equilibration of small subsystems are indeed short **[A]**.

We also focus on measurements in quantum mechanics in the context of decoherence of states. Decoherence is used to explain and study the quantum-to-classical state transitions. One of the features of the decohered (classical) state is its robust, objective character, contrasting the delicate nature of quantum systems. Here we analyze in a model-independent way, by considering random Hamiltonians, a measurement process by studying the timescales of decoherence, showing that objective results typically appear in quantum measurements, provided the systems involved are large-dimensional and we wait long enough **[B]**.

In the end we focus on accessibility of our results, by checking how the Haar measure can be implemented by local quantum circuits. Namely, we focus on, so-called, approximate unitary t-designs which is a distribution of unitaries which mimic properties of the Haar measure for polynomials of degree up to t. It is know that random quantum circuits are approximate unitary t-designs, but not a lot is know about the length of the circuit needed to form the t-design. We numerically investigate these relations by evaluating spectral gaps that give a ratio of convergence to a given t-design **[C]**.

Part 2:
What does thermodynamics look like in the absence of the thermodynamic limit? In the second part of the thesis we focus on the question *what does thermodynamics look like in the absence of the thermodynamic limit?* In recent years there has been a concerted effort to apply techniques from quantum information theory to study the laws of thermodynamics at the nano and quantum scale. This has led to the resource theory of Thermal operations for determining when single-copy transformations, of small systems in the presence of a large heat bath, are possible. However, most results has been obtained for states that are diagonal in energy-basis. To have truly quantum limitations for state transitions, one needs to check how fully quantum states, i.e. those having coherences behave. We obtained thermodynamic laws that are unique to quantum systems in a superposition of states by investigating transformations between different matrix elements, representing quantum states. Specifically, we have provided necessary and sufficient conditions for transition between qubit states under so-called Thermal operations **[D]**.

Nonetheless, if one wants to experimentally manipulate a thermodynamical system, implementing an arbitrary thermodynamical operation which controls all microscopic degrees of freedom is infeasible in the foreseeable future, especially if the operation requires control over microstates of the heat bath. Here we show that arbitrary operations between the system and bath can be replaced by two very simple operations, namely, changing the energy levels of the system and thermalizing any two system energy levels. What is more, using only these two operations and one ancilla qubit in a thermal state, one can transform any state into any other state allowable by the second laws. This brings the full array of Thermal operations into a regime more feasible by experiment, and allows for thermodynamical transformations of a state which have never thus far been performed **[E]**.

This thesis contains results obtained in the following publications and preprints:

[A]: F.G.S.L. Brandão, P. Ćwikliński, M. Horodecki, P. Horodecki, J. Korbicz, and M. Mozrzymas *Convergence to equilibrium under a random Hamiltonian*, Phys. Rev. E 86, 031101 (2012);

[B]: J. K. Korbicz, E. A. Aguilar, P. Ćwikliński, and P. Horodecki, *Do objective results typically appear in quantum measurements?*, arXiv:1604.02011 (2016);

[C]: P. Ćwikliński, M. Horodecki, M. Mozrzymas, Ł. Pankowski, and M. Studziński, *Local random quantum circuits are approximate polynomial-designs - numerical results*, J. Phys. A: Math. Theor. 46, 305301 (2013);

[D]: P. Ćwikliński, M. Studziński, M. Horodecki, and J. Oppenheim, *Limitations on the evolution of quantum coherences: Towards fully quantum second laws of thermodynamics*, Phys. Rev. Lett. 115, 210403 (2015);

[E]: C. Perry, P. Ćwikliński, J. Anders, M. Horodecki, and J. Oppenheim, *A sufficient set of experimentally implementable thermal operations*, arXiv:1511.06553 (2015).