Appendix no. 4

SELF-PRESENTATION

1. Name: **Piotr Józef Bartłomiejczyk**

2. Obtained diplomas and degrees:

- M.Sc. in Mathematics, University of Gdańsk, Faculty of Mathematics, Physics and Chemistry, 1991;
- Ph.D. in Mathematics, Dissertation: *Topics in Connection Matrix Theory* (advisor prof. dr hab. K. Gęba), Institute of Mathematics, Polish Academy of Sciences, Warsaw, Poland, 2000.

3. Employment in scientific institutions:

- Assistant, Institute of Mathematics, University of Gdańsk, 1991–1999,
- Assistant Professor, Institute of Mathematics, University of Gdańsk, 2000–2011,
- Senior Lecturer, Institute of Mathematics, University of Gdańsk, 2011–2014,
- Senior Lecturer, Department of Differential Equations and Mathematics Application, Gdańsk University of Technology, from 2014.

4. Scientific achievements resulting from Article 16 Paragraph 2 of the Act on Academic Degrees and Titles and on Degrees and Titles in the Field of the Arts of 14 March 2003 is a series of publications under the title:

"Homotopy properties of the space of local maps"

List of publications included in the above mentioned achievement:

[H1] **P. Bartłomiejczyk**, K. Gęba, M. Izydorek, *Otopy classes of equivariant local maps*, J. Fixed Point Theory Appl. 7(1) (2010), 145–160.

[H2] **P. Bartłomiejczyk**, P. Nowak-Przygodzki, *Gradient otopies of gradient local maps*, Fund. Math. 214(1) (2011), 89–100.

[H3] **P. Bartłomiejczyk**, P. Nowak-Przygodzki, *Proper gradient otopies*, Topol. Appl. 159 (2012), 2570–2579.

[H4] **P. Bartłomiejczyk**, P. Nowak-Przygodzki, *The exponential law for partial, local and proper maps and its application to otopy theory*, Commun. Contemp. Math. 16(5) (2014), 1450005 (12 pages).

[H5] **P. Bartłomiejczyk**, P. Nowak-Przygodzki, *On the homotopy equivalence of the spaces of proper and local maps*, Cent. Eur. J. Math. 12(9) (2014), 1330–1336.

[H6] **P. Bartłomiejczyk**, *On the space of equivariant local maps*, Topol. Methods Nonlin. Anal. 45(1) (2015), 233–246.

[H7] **P. Bartłomiejczyk**, P. Nowak-Przygodzki, *The Hopf theorem for gradient local vector fields on manifolds*, New York J. Math. 21 (2015), 943–953.

Below is a discussion of the scientific aim of the above mentioned achievement.

1. Discussion of the results of the monothematic series of publications [H1]–[H7]

1.1. **Introduction.** The subject of works that make up the dissertation has its source in two currents of research. The first one concerns spaces of local maps and otopies and the second one is related to looking for new topological invariants in the class of gradient maps and homotopies.

The idea of studying spaces of partial, local and proper maps comes from [1, 26, 27, 33, 42]. The paper [1] by A. M. Abd-Allaha and R. Brown from 1980 is the oldest and most elementary of them. The authors introduced there the space of partial maps $\mathrm{Par}(X, Y)$, where X, Y are topological spaces. This space consists of continuous maps $f\colon U \subset X \to Y$ defined on open subsets $U \subset X$ and its topology is a version of compact-open topology adapted to changing domains. Since, as it is easy to see, the above space is contractible if Y is contractible, not the whole space but its subsets consisting of local and proper maps have been used in nonlinear analysis. These subsets are also topological spaces but their topologies are essentially finer than the topology induced from the space of partial maps. Both spaces owe their usefulness to these topologies.

The space of proper maps appears in the paper [27] by J. C. Becker and D. H. Gottlieb from 1999. The topology in the set of local maps was introduced in our paper [21] and then in full generality in [H5]. It should be emphasized that in [H5] we introduce a generalized definition of a local maps that includes both local maps in the old sense and proper maps.

However, much earlier than we managed to define the topology in the set of local maps the notion of a local maps and a very useful generalization of the concept of homotopy called otopy have been introduced and used in the papers by J. C. Becker and D. H. Gottlieb ([26]) and D. H. Gottlieb and G. Samaranayake ([42]). The main advantage of using these notions is that otopy relates local maps with not necessarily the same domain, because the domain of a map may change along otopy. What is important is that the topological degree is otopy invariant and otopy classes appear naturally in many classification results.

The second important inspiration of the works making up the dissertation is the study of gradient maps and homotopies, in particular, the article [52] by A. Parusiński from 1990, which is closely related to discoveries made in the previous decade. Namely, in the middle of eighties E. N. Dancer gave a definition of a new degree-type invariant for S^1-equivariant gradient maps ([32]). Since this new degree provides more information than the usual degree, one can obtain new bifurcation results.

In the eighties of the last century Prof. K. Gęba posed the following problem: is there a better invariant for gradient homotopies of gradient maps than the usual topological degree? In 1990 A. Parusiński [52] gave the negative answer. Namely, he proved that if two gradient vector fields on the unit disc D^n and nonvanishing in S^{n-1} are homotopic (have the same degree), then they are gradient homotopic.

It occurs that the problem posed by Prof. K. Gęba appears naturally if we consider local maps and their otopy classes. For that reason analysis and comparison of gradient and usual otopy classes occupies an important place in our research.

It is worth pointing out that independently of Becker and Gottlieb, the similar notion was also developed by Dancer, Gęba and Rybicki in the article [33] from 2005. Understandably, they use different terminology. Local maps are called compact pairs (a pair consists of a map and its domain) and otopies are called homotopies of compact pairs. The authors use these notions as tools for proving results on the classification of equivariant gradient otopy classes.

All papers included in the dissertation concern the space of local maps and their various subspaces (with the induced topology) consisting of gradient or equivariant maps. We focus on the study of otopy classes of local maps i.e. path-components of the above spaces. We also show classifications of various sets of otopy classes (usual, gradient, equivariant) and natural relations between different sets of otopy classes.

For the sake of clarity, we will divide our discussion into three parts. In the first part we focus on the study of the set of gradient otopy classes. The main results of this part concern the set of gradient local maps in $\mathbb{R}^n$ ([H2]), the set of proper gradient maps in $\mathbb{R}^n$ ([H3]) and the set of gradient local vector fields on a manifold ([H7]). The main topic of the second part is an introduction of the topology in the set of local maps, which allows us to interpret otopies as paths in the space of local maps (similarly as for homotopies) and establish the relation between the theory of otopy and homotopy. In this part we also explain the relation between the space of proper maps and the space of local maps (in the narrower sense) if we restrict ourselves to the Euclidean case. In turn in the third part we deal with equivariant local maps and their otopies. We present here a version of the equivariant degree theory formulated in the language of otopies ([H1]) and results concerning the decomposition of the set of equivariant otopy classes with respect to set of orbit types ([H6]).

Let us mention that problems of that type seems to be quite natural. G. Segal proves in [58] that inclusions of some function spaces are homotopy (homology) equivalences. Similar results are contained in papers on configuration spaces by G. Segal and D. McDuff (see [50, 59]). On the other hand M. Gromov in his book ([43]) outlines a program of research on relations between a space of all maps and its subspaces given by some partial differential relations. Let us note that Schwarz condition (being such a differential relation) is equivalent to the statement that a map is gradient.