The assistant members of the Organizing Committee - Ph.D. students: W. Żwirbla and P. Borysewicz.

The group of Polish participants during the banquet.
From the left: M. Aleksiejuk, F. Rejmund, Z. Ranachowski.

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Optical gaussian beam in acoustooptics. Theoretical description of noncollinear isotropic interactions

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ABSTRACT

Acoustooptical phenomenon was anticipated theoretically by L. Brillouin in 1922. These anticipations were confirmed experimentally 10 years later. In the following years the influence of elastic wave on light wave propagation was described from many points of view, examined experimentally, and applied in practice. The first works describe interaction of unlimited plane waves but the fundamental meaning for practical applications has the description of real wave beams' interaction. This problem is still existing despite huge progress made in acoustooptical phenomenon's investigations. Among numerous description methods applicable to various, specific acoustooptical interactions are also such ones, that make it possible to describe interaction of beams of arbitrary spatial distributions. However, in today's applications of acoustooptical phenomenon such as modulators, filters, spectrum analyzers, etc., one uses mostly laser light. Light beams emitted by laser may in many cases be described as gaussian beams. It means, that the correct description of acoustooptical interaction involving optical gaussian beams has significant practical meaning. This paper presents the review of theoretical works describing acoustooptical interaction between optical gaussian beams and acoustical waves. Special attention has been paid to the application of complex geometrical optics' methods.

Keywords: Raman-Nath diffraction, geometrical optics, complex geometrical optics, gaussian light beams, light beam deflection, light beam phase change, geometrical optics of nonhomogeneous media, perturbation calculus

1. INTRODUCTION

For acoustooptical interactions many effective methods for wave equation approximate solution were worked out, especially for different particular cases, as e.g. Raman-Nath interaction or Bragg interaction. These methods have been applied already in first works on this topic, e.g.[1, 2] and went into acoustooptics theoretical grounds and it may be found in many school-books[3, 4, 5].

Independently, the geometrical optics methods were also used, e.g. in one of the first works of Lucas[6] at the initial stage of the evolution of acoustooptics. That work and other similar to that one are discussed in books[7, 8]. Because of wave theory successes, the methods of geometrical optics were forgotten for many years. But in last years, thanks to important progress in geometrical optics, these methods become competitive relatively to "exact" wave optics methods, especially for problems, in which approximate methods for wave equation solution are used (because of various reasons).

The mentioned above first works[1, 2, 6] describe interaction of unlimited plane waves but the fundamental meaning for practical applications has the description of real wave beams' interaction. This problem is still existing in present literature about acoustooptical phenomenon. Because in today's applications of acoustooptical phenomenon, such as modulators, filters, spectrum analyzers, etc., one uses mostly laser light, the correct description of acoustooptical interaction involving optical gaussian beams has significant practical meaning.

This paper presents the review of theoretical works describing acoustooptical interaction between optical gaussian beams and acoustical waves. Special attention has been paid to the application of complex geometrical optics' methods. In presented work, this method for Raman-Nath acoustooptical interaction description was used. It is based on the geometrical optics fundamental equations for homogeneous and nonhomogeneous media[9, 10, 11]. Next, these equations (especially the ray perturbation calculus method) are used for describing gaussian beam propagation in the isotropic

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medium (water, liquid) with an column disturbed by the plane ultrasound wave for the case when both waves run perpendicularly to each other. Presented work is an extension of the previous ones[12, 13].

2. ACOUSTOOPTICAL INTERACTION EQUATIONS

Propagation of light waves in a optically nonhomogeneous and non-absorbing medium is described by the equation[3, 4]

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\left(\hat{\varepsilon}\vec{E}\right)=\Delta\vec{E}-\vec{\nabla}\left(\vec{\nabla}\circ\vec{E}\right), \tag{1}$$

where c is speed of light in the vacuum, $\vec{E}$ is intensity of the electric field of the wave, and $\hat{\varepsilon}$ is the medium permittivity tensor. This equation is useless in practice[14]. It follows from the equations (constitutive and Gauss law)

$$\vec{D}=\varepsilon_0\hat{\varepsilon}\vec{E}\,,\quad \vec{\nabla}\circ\vec{D}=0 \tag{2}$$

that in optically nonhomogeneous media, in isotropic media and in media devoid of free charges, the second component on the right side of the formula (1) disappears. In nonhomogeneous media (media in which acoustic wave is propagate are always nonhomogeneous) it can be indicated[3] that this complement is proportional to $\varepsilon_1\lambda/\lambda_a << 1$ where ε_1 is the amplitude of permittivity changes and λ and λ_a are in order the lengths of light and acoustic wave. In a such situation the considered component of the equation (1) can be omitted so the propagation of light waves in the acoustooptic interaction is described by the following equation

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\left(\hat{\varepsilon}\vec{E}\right)=\Delta\vec{E}\,. \tag{3}$$

In the above - mentioned equation, it is possible to include wave suppression if permittivity is assumed to be a complex. However acoustooptic interaction is usually carried out in media of a good optical quality in which light wave attenuation is omitted.

Propagation of elastic waves in crystals is described by the equation[4, 15, 16]:

$$\vec{\nabla}\circ\boldsymbol{\sigma}^T=\rho\frac{\partial^2\vec{u}}{\partial t^2}-\boldsymbol{\eta}:\frac{\partial}{\partial t}\left(\vec{\nabla}\boldsymbol{u}\right), \tag{4}$$

where $\boldsymbol{\sigma}$ is the stress tensor, $\boldsymbol{\eta}$ the viscosity tensor, $\boldsymbol{u}$ the strain tensor, $\vec{u}$ the displacement vector and ρ medium density. The index ^{T} stands for transposing. The second complement on the right side of the equation (4) describes elastic wave attenuation. Although light waves attenuation in the acoustooptic interaction can be generally omitted, acoustic wave attenuation can be significant in many cases.

The acoustic wave propagating in a medium by means of photoelastic phenomenon induces changes of permittivity tensor. These changes can be shown as follows[15, 17]

$$\boldsymbol{\kappa}_1=\boldsymbol{P}:\left(\vec{\nabla}\vec{u}\right), \tag{5}$$

where

$$\hat{\boldsymbol{\kappa}}=\boldsymbol{\kappa}+\boldsymbol{\kappa}_1\,,\ ,\ \hat{\boldsymbol{\kappa}}=\hat{\boldsymbol{\varepsilon}}^{-1}\,,\quad \boldsymbol{\kappa}=\boldsymbol{\varepsilon}^{-1}\,,\quad \hat{\boldsymbol{\varepsilon}}=\boldsymbol{\varepsilon}+\boldsymbol{\varepsilon}_1\,,\quad \boldsymbol{\varepsilon}_1\cong-\boldsymbol{\varepsilon}\,.\,\boldsymbol{\kappa}_1\,.\,\boldsymbol{\varepsilon}\,. \tag{6}$$

Here $\boldsymbol{\varepsilon}$ and $\boldsymbol{\kappa}$ stand for electrical permittivity and unpermittivity tensors of the medium undisturbed by a acoustic wave. Furthermore $\boldsymbol{P}$ stands for the photoelastic constant tensor which in a general case includes direct effect, indirect one (through electrooptic and piezoelectric effects) and the influence of local rotations of the medium[15, 17]. The last equation in (6) is valid when changes of both tensors are very small.

The above-mentioned system of equations (3) - (6) as usual should be completed with adequate boundary conditions. If one is interested only in steady-state solutions, initial conditions are unimportant. The general solution for this system of equations is out of the question because of its complication. Anyway, that solution wouldn't be too useful. Usually what we search for are the solutions for specific application. A few groups of problems, including different features, can be distinguished. Division of acoustooptic interactions on interactions Raman-Nath and Bragg types belongs to the basic ones. Generally it can be said that the first ones occur for acoustic waves of respectively low frequency (up to some MHz) and/or for short paths of interaction and the second ones for very high frequency (over a few dozen MHz) and/or for long paths of interaction. It follows that there is a intermediate zone between these two kinds of interaction. It was a subject of intensive theoretical and experimental studies. Furthermore, due to the condition of light wave polarization,

acoustooptic interaction divides into isotropic (without polarization changing) and anisotropic (changing the state of polarization).The latter are characteristic for acoustooptic interactions in optically anisotropic media and/or for interactions with a transverse acoustic wave. Collinear and non-collinear interactions are also distinguished. The first ones occurs when the acoustic beam and the light beam run parallel to each other in the interaction zone. In the second case both beams intersect at an sizeable angle, often close to the right angle.

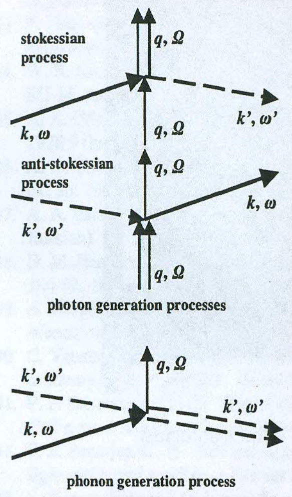

Fig. 1 Diagrams for description of quantum processes in acoustooptical interaction. k, k', ω, ω' are wave vectors and circular frequencies of photons and q, Ω are wave vector and circular frequency of phonons respectively.

Geometry of acoustooptic interaction is well expressed by the quantum image of this interaction. The light and acoustic beams can be depicted as in order fluxes of photons and phonons. It means that acoustooptic interaction follows from photon-phonon interaction. Exemplary schemes of this interactions are demonstrated in the Fig. 1. They can be divided into two groups – processes with generation of photon and processes with generation of phonon. The first ones are very important in every acoustooptic interaction. The second ones can be omitted if light intensities are standard. The reason of this is considerably small density of photon streams in the light beam in comparison to density of phonon stream of the acoustic beam. In such a situation it can be assumed that the acoustic wave in the acoustooptic interaction practically doesn't change. The processes of photon production are important when gigantic light impulses interact.[18, 19]. In every photon-phonon interaction principles of conservation of momentum and energy must be fulfilled:

$$\vec{k}=\vec{k}'+\vec{q}\,,\quad \omega=\omega'+\Omega\,. \tag{7}$$

Relations (7) with dispersion laws for light and acoustic waves define geometry of the acoustooptic interaction.

Schemes shown in the Fig. 1 are characteristic for optically and acoustically isotropic and undispersive media. Regarding that in a such case we have $k\approx k'>>q$, photon „incidence" angle on the phonon direction amounts to

$$\Theta_B\cong\frac{q}{2k}=\frac{\lambda}{2n\lambda_a}, \tag{8}$$

and is called Bragg angle. Here λ is the light wavelength in the vacuum and n is the medium refractive index. It follows from the quantum image that the acoustooptical interaction of plane waves occurs only at a Bragg angle. In waves that are not plane there is a spatial spectrum of wave vectors and acoustooptical interaction of such waves can occur at „many" angles. What's more, in a such case the beams created as a result of the interaction can again fulfill the conditions of matching (7), and as a result of this next light beams can appear. This mechanism is the basis of many methods of solving the system of equations (3) - (6). The solution is presented in the form of the sum of waves whose frequencies and wave vectors results from multiple usage of conditions (7), while the beam that results from m-multiplicity photon-phonon interaction is conjugated with only two adjacent beams: $m-1$ i $m+1$.

For example, let's consider the easiest version[7] of Raman-Nath plane wave interaction: the light one of frequency ω and of the wave vector $\vec{k}$ propagated along the axis OZ and the acoustic one of the frequency Ω and the wave vector $\vec{q}$ propagated along the axis OX in the isotropic medium. The acoustic wave of low intensity causes mainly the change of the light wave phase (deflection of the light beam is neglected)

$$\Delta\varphi(x,z)=k\Delta n\cos(\Omega t-qx)\left[(z-z_l)H(z-z_l)-(z-z_p)H(z-z_p)\right]. \tag{9}$$

It has been assumed that the acoustic beam is propagate in the zone of the width $\Delta z=z_p-z_l$ ($H(z)$ stands for the Heaviside step function) and Δn is the amplitude of medium refraction coefficient changes caused by the acoustic wave. From relation (6) results that Δn is proportional to wave number and amplitude (power) of acoustic wave. In this situation, after going through the interaction zone, intensity of the light wave electric field looks like

$$E_y=E_{y0}\exp\{i[\omega t-kz+p_{RN}\cos(\Omega t-qx)]\}=E_{y0}\sum_{-\infty}^{+\infty}i^mJ_m(p_{RN})\exp\{i[(\omega+m\Omega)t-(kz+mqx)]\}, \tag{10}$$

where J_m stands for the ordinary Bessel function of the order m, E_{y0} is the amplitude of electric field intensity and $p_{RN} = k\Delta n\Delta z$ is the so called Raman-Nath parameter. In the statement (10) properties of generating function for Bessel function[20] were used. Physically it is right for not big quantities of Raman-Nath parameter. It demonstrates an infinite set of beams diffracting on the acoustic wave, having the frequencies $\omega_m = \omega + m\Omega$ and propagated at an angle $\mathrm{tg}(\alpha_m) = mq/k$ relatively to axis OZ. It follows from the Bessel function properties that the diffraction image is symmetrical to the plane XY. It should be emphasized that in the case of the assumed geometry of interaction, only the light wave polarized along the axis OY doesn't change its polarization (isotropic interaction), what is clearly showed in relation (10).

When the light and acoustic beams interact at the angle that isn't the right angle (for the incidence angle Θ), a asymmetry appears in the diffraction image. We may take this fact into consideration by means of the mechanism described above which formally means generalization the solution (10) to the form:

$$E_y = \sum_{-\infty}^{+\infty} E_m(z)\exp\left[i\left(\omega_m t - \vec{k}_m \circ \vec{r}\right)\right], \quad \vec{k}_m = \left[k_x + mq, 0, k_z\right], \quad \vec{k} = \left[k\sin\Theta, 0, k\cos\Theta\right], \tag{11}$$

where the amplitudes E_m should be calculated from the condition that this solution must fulfill the equation (3). It means that if we assume that the amplitudes of the diffracted beams are slowly varying function in space, i.e. if $\partial^2 E_m/\partial z^2 << k(\partial E_m/\partial z)$, then they must fulfill the infinite system of equations

$$\frac{dE_m}{dz} - 2ib_m E_m = -iw_{mm}\left(E_{m-1} + E_{m+1}\right), \tag{12}$$

where

$$E_m(z = z_l) = E_{y0}\delta_{m0}, \quad b_m = \frac{k_m^2 - k_{Bm}^2}{4k_z}, \quad w_{mn} = \frac{\varepsilon_{1a}}{\varepsilon}\frac{k_{Bn}k_{Bm}}{4k_z}, \quad k_{Bm}^2 = \frac{\varepsilon\omega_m^2}{c^2}. \tag{13}$$

In the latter formulas δ_{mn} denote the Kronecker delta and $\varepsilon_{1a} \approx 2n\Delta n$ is the amplitude of medium permittivity changes. In the first approximation two equations – for V_0 and V_{+1} or for V_0 and V_{-1} – should be left from the system of equations (12). These two equations may be solved in the approximation of so called the given pumping field amplitude which gives

$$E_0 \cong E_{y0} = const, \quad E_{\pm 1} = iE_{y0}w_{\pm 1 \pm 1} z \exp(ib_{\pm 1}\Delta z)\mathrm{sinc}(b_{\pm 1}\Delta z), \tag{14}$$

or we may include the changes of intensity in both beams which gives

$$E_0 = E_{y0}\exp(ib'_{\pm 1}\Delta z)\sqrt{1 - w_{0\pm 1}^2(\Delta z)^2\left[\mathrm{sinc}\left(\sqrt{b_{\pm 1}^2 + w_{0\pm 1}^2}\,\Delta z\right)\right]^2}, \quad E_{\pm 1} = iE_{y0}w_{\pm 1 \pm 1}\Delta z \exp(ib_{\pm 1}\Delta z)\mathrm{sinc}\left(\sqrt{b_{\pm 1}^2 + w_{0\pm 1}^2}\,\Delta z\right). \tag{15}$$

In the formulas (14) and (15) $\mathrm{sinc}(u) = \sin(u)/u$, $b'_{\pm 1} = b_{\pm 1} - (1/z)\mathrm{arctg}\left[b_{\pm 1} z\,\mathrm{tg}\left(\sqrt{b_{\pm 1}^2 + w_{0\pm 1}^2}\,z\right)\Big/\left(\sqrt{b_{\pm 1}^2 + w_{0\pm 1}^2}\,z\right)\right]$. It follows from both expressions on $V_{\pm 1}$ that they reach the maximal value when $\Theta \approx \Theta_B$, so when the matching condition is fulfilled. Furthermore, in the second case depending on the length of the interaction distance z we observe mutual pumping over energy from the incident beam to the diffracted beam and back. Presented expressions shows so called Bragg diffraction. They are right when light intensities in higher diffractional orders can be omitted. They can be included by taking into consideration more equations from the system (12).

The solutions of the equations of acoustooptics (3) - (6) define distribution of light wave electric field at output from the interaction zone. Propagation of light beams outside this zone demands further analysis. Here one uses the theory of electromagnetic wave diffraction e.g. Kirchhoff integral formula[3,14]. One may also directly formulate the acoustooptical interaction problem in the language of integral equations e.g. [21,22]. It makes it possible to achieve the expressions for distribution of the light waves' electric field directly in the zone of detection (after interaction with an acoustic wave). These methods will not be analyzed in the following part of the paper.

Presented methods of describing acoustooptical interaction are characterized by a kind of inconsistency. According to the quantum image, plane waves' interaction can occur only when matching conditions are fulfilled i.e. when the beams interact at the Bragg angle. However it follows from these descriptions that in the case of Raman-Nath diffraction multiple interactions appear and in the case oh the Bragg diffraction – interaction occurs also at angles that differ from the Bragg angle. Calculated results are the consequence of the applied simplifications that are physically well justified.

It should be added these results have been experimentally well confirmed. However a more correct theoretical description should include the fact that all acoustooptically interacting wave beams are spatially bounded beams.

3. ACOUSTOOPTICAL INTERACTION OF BOUNDED BEAMS

It is obvious that the bases of the description of real wave beam interaction are methods of the theoretical description of the limited in space and time beams. Limits in time are significant in pulse interaction research and are not considered in this paper. The basic instrument used to describe of spatially bounded beams is the Fourier analysis. Another important instrument is the ray methods, i.e. geometrical optics methods in general.

4.1. Fourier transform methods

One of the first works devoted to interaction optical gaussian beam with acoustic wave is Hargrove's work. In that work, the author considered standard Raman-Nath interactions with a plane-parallel acoustic beam, which runs orthogonal to gaussian optical beam in the place of its narrowing. Formally, the acoustic beam could have any time run, but the final results were given for a homogenous harmonic wave. To determine the distribution of the electric field a diffractional integral was used. (comp. [3])

$$E = E_{y0}\int_{-\infty}^{+\infty}\exp\left[i(k_x x - \omega t)\right]\exp\left[i\upsilon(\varphi_a(t))\right]\exp\left(-\frac{x^2}{2a^2}\right)dx, \quad k_x = k\sin\Theta, \quad \upsilon(\varphi_a(t)) = q\Delta zn(\varphi_a(t)), \quad \varphi_a(t) = qx - \Omega t. \tag{16}$$

The υ function describes the phase change of the light wave caused by the changes of the refractive index n, which were produced by the acoustic wave. This function is often called a transfer function, and the method itself – a transfer function analytical method (formalism). The a parameter is the intensity ray of the gaussian beam in the narrowing, and the $\varphi_a(t)$ is the phase of the acoustic wave. By using the Fourier transform

$$\exp\left[i\upsilon(\varphi_a(t))\right] = \sum_{-\infty}^{+\infty}\Phi_m \exp(im\varphi_a(t)) \tag{17}$$

we obtain

$$E = E_{y0}\sum_{-\infty}^{+\infty}\Phi_m W_m \exp\left[-i(\omega + m\Omega)t\right], \tag{18}$$

$$W_m = \exp\left[-\frac{1}{2}(k_x + mq)^2 a^2\right] = \exp\left[-(\pi G(H + m))^2\right], \quad G = \frac{\sqrt{2}a}{\lambda_a}, \quad H = \frac{\lambda_a}{\lambda}\sin\Theta. \tag{19}$$

W_m is called overlap factor. When $G \to \infty$ (wide gaussian beam), the parameter W_m is significantly different from zero only when $(H + m) \to 0$, so for $\sin\Theta = -m\lambda/\lambda_a$ (matching condition, Bragg's angles). That means that we deal then with "discrete" diffractive orders. When $\upsilon(\varphi_a(t)) = p_{RN}\sin(\varphi_a(t))$, then while counting inverse Fourier transform we obtain

$$\Phi_m = \frac{1}{\pi}\int_0^{\pi}\cos(p_{RN}\sin\varphi_a - m\varphi_a)d\varphi_a = J_m(p_{RN}) \tag{20}$$

(like in (10)). When $G \approx 1$ ("narrow" gaussian beam) more or less continuous (distinct from the "discrete" one) light intensity distribution, which can be calculated by time averaging $EE^* = |E|^2$, where E is described by the formula (18). The calculations predict that the diffracted beams get wider, and even partially overleap themselves.

It must be emphasized that the problem of the narrow gaussian beam interaction in the Raman-Nath field is still current. Ohtsuka and Tanone's[24] work is devoted to this problem. From theoretical part this work brings no new elements, but there are given comparisons of calculation results with measurements, which show good conformity. One of the latest works devoted to this problem is Windels and Leroy's[25] work. The authors used a decomposition of falling gaussian beam into Fourier spacious components. It was assumed, that the narrowing of this beam is in the entrance $z_l = (0)$ into the field of interaction with plane-parallel acoustic beam. Next, for each of the components a formula (given in the book[26]) for a phase change in a continuous acoustic wave field was used. In the exit from the acoustooptical interaction field, the changed Fourier components were put together into a resultant exit beam described by the formula:

$$E(x, z = z_p, t) = E_{y0}e^{i\omega t}\int_{-\infty}^{+\infty}dk_x \exp\left(-ik_x x - ik_z z_p - k_x^2 a^2\right)\exp\left\{-i\frac{k^2 n}{k_z}\left[\Delta n\Delta z\,\mathrm{sinc}\left(\frac{qk_x\Delta z}{2k_z}\right)\sin\left(\frac{qk_x\Delta z}{2k_z} - \varphi_a(t)\right)\right]\right\} \tag{21}$$

To calculate this integral, the phase formula in the last exponent superscript was developed, with the accuracy to the k_x^2 proportional terms, into a series. The free term of this development gives results conforming with the ones obtained in the work[23]. Taking other components into consideration gives:

$$E(x, z = z_p = \Delta z, t) =$$

$$E_{y0}\frac{\exp\left[i\left(\omega t - kn\Delta z + k\Delta n\Delta z\sin\varphi_a(t)\right)\right]}{\sqrt{2\pi}\sqrt{a^2 - i\left[\frac{\Delta z}{k} - \frac{4}{3}k\Delta n\Delta z\left(\frac{q\Delta z}{2k}\right)^2\sin\varphi_a(t)\right]}}\exp\left\{-\frac{\left[x + \frac{q\Delta n(\Delta z)^2}{2}\cos\varphi_a(t)\right]^2}{2\left[a^2 - i\left(\frac{\Delta z}{k} - \frac{4}{3}k\Delta n\Delta z\left(\frac{q\Delta z}{2k}\right)^2\sin\varphi_a(t)\right)\right]}\right\} \qquad (22)$$

In this formula significant changes in shape of gaussian beam envelope can be noticed. It can be seen, that the beam center position is modulated with the acoustic wave frequency. The light beam deflection in the acoustic wave field is responsible for this effect. It can be also noticed, that the beam width becomes a complex value. It causes also a modulated beam widening (in comparison to its diameter at the entry) and an additional displacement of its phase. The light beam width changes were described as focusing and defocusing of that beam. These effects in the discussed work[25] were illustrated with graphs and were compared with the work[23] results. It should be added, that in the measurements of Raman-Nath interactions, the modulation of the light beam placement as a result of its deflection in the acoustic wave field, usually is not registered. It demands the use of special measurement techniques. Usually a time average light beam intensity in a given place of detection plane, normally putted in far field region, is measured. In the discussed work, the far field was obtained by the Fourier transform of formula (23). The comparison of light intensity distributions in the examined beam, obtained by this method, with the distributions obtained in the work[23] shows that according to the new model, the beams are wider and of smaller maximum intensity.

One of the first works devoted to Bragg's interactions with the optical gaussian beam participation analysis is Magdich and Molchanov's[27] work. The authors used a method similar o the one used in the work[25] (transfer function formalism). The case when the optical beam axis falls on the plane-parallel homogeneous acoustic beam at the Bragg's angel was discussed. The optical gaussian beam (gaussian only in one dimension, that is cylindrical) falling on the column of ultrasounds was, by Fourier transform, decomposed into its spacious spectrum. Each component of this spectrum interacts regardless of the others with homogenous and plane acoustic wave, and as a result of this interaction the components change according to the expression (14). All the component spectrums of the deflected beam, after leaving the field of interaction were composed into output light beam (in the "exit" from the interaction field) by using an appropriate Fourier retransform. The final results were described as a function of the parameter that is a discrepancy of angular light and acoustic beams ratio: $d_a = [\lambda/(\sqrt{2}\pi a)]/[\lambda_a/(2\Delta z)]$. These results depend also on the acoustic beam power, which is on the acoustooptical interaction "power". For the weak interaction and $a \to \infty$ (the wide light beam, $d_a \to 0$) the courses just like for the plane waves (comp. (14) and (15)) are obtained. When with a weak interaction we have $d_a >> 1$, then the Bragg's diffraction efficiency establishes on a constant level, because only a part of the light beam interacts with the acoustic beam (only a smaller and smaller "part" of the light beam fulfills the fitting conditions (7)). With a strong interaction energy pumping over between the falling and deflected beams occurs, however, the whole energy pumping over is impossible (like it is predicted in the formula (15)). In their next work[28], the authors generalized the results for the case of two-dimensional gaussian beams and they gave formulas for the deflected beam field in a far zone.

A special place in the description of the optional amplitudes distribution acoustooptical wave beams belongs to the works of A. Korpel and his associates[29-32]. In those works , just like in the previous ones, the dissipation in the plane XY was considered, but for the first time an optional field distribution also in the acoustic beam. However, traditionally neglected were: acoustic waves damping and electric field changes caused by the acoustooptical interaction. The base of the analysis is the system of equations (12), in which the dependence of the acoustic field amplitude on the placement in the beam must be taken into consideration. That dependence occurs in the formula (6). The calculations take place in the Fourier space, but:

$$u(\vec{\rho}) = \int\tilde{u}(\vartheta)\exp(-i\vec{q}\circ\vec{\rho})d\vartheta, \quad E_m(\vec{\rho}) = \int\tilde{E}_m(z,\Theta)\exp\left(-i\vec{k}\circ\vec{\rho}\right)d\Theta, \quad E_{y0}(\vec{\rho}) = \int\tilde{E}_{y0}(z,\Theta)\exp\left(-i\vec{k}\circ\vec{\rho}\right)d\Theta, \qquad (23)$$

where $\vec{\rho} = [x, z]$, $\Theta = \angle(\vec{k}, OZ)$, $\vartheta = \angle(\vec{q}, OX)$. In the expression (23) wave vector changes in deflected beams were neglected. That is correct for not too high diffractive orders, not too high acoustic waves frequencies and for the optically isotropic mediums. These assumptions are fulfilled in so called small-angular acoustooptical interaction. In that case the system of equations (12) looks like this:

$$\frac{\partial\tilde{E}_m(z,\Theta_m)}{\partial z} = \frac{-ikC}{4\cos(\Theta_{m-1})}u(z, x_{m-1})\tilde{E}_{m-1}(z,\Theta_{m-1}) + \frac{-ikC}{4\cos(\Theta_{m+1})}u^*(z, x_{m+1})\tilde{E}_{m+1}(z,\Theta_{m+1}), \qquad (24)$$

where

$$\sin\Theta_m = \frac{k_{mx}}{k} \equiv \Theta_m, \quad \sin\Theta_{m\pm1} = \frac{k_{mx} \pm q_x}{k}, \quad \Theta_{m\pm1} = \Theta_m \pm 2\Theta_B, \quad x_{m\pm1} = z\tan(\Theta_m \pm \Theta_B). \qquad (25)$$

Moreover $C = -\varepsilon p$, where p is an adequate photoelastic constant, k_{mx} is the OX component of the wave vector in m diffractive order, and * index is a complex conjugate value. When the equation (24) is integrated by z, we obtain

$$\tilde{E}_m = \delta_{m0}\tilde{E}_{y0} - ia_{m-1}\int_{-\infty}^{z}u_{m-1}^{+}\tilde{E}_{m-1}dz - ia_{m+1}\int_{-\infty}^{z}u_{m+1}^{-}\tilde{E}_{m+1}dz, \qquad (26)$$

where in order to simplify, the arguments of the amplitudes were neglected and new designations were added:

$$a_m = \frac{kC}{4\cos(\Theta_m)}, \quad u_m^{+} = u(z, x_{m+1}), \quad u_m^{-} = u^*(z, x_{m-1}). \qquad (27)$$

The advantage of the expressions (26) is , that it is possible to use them recurrently. This method is typical for multiple scattering analysis. The determination of the amplitude in the m diffractive order requires the calculation of the shares resulting from scattering running in all possible "trajectories" from the falling beam (E_{y0}) to the m beam (that means optional quantity through all other beams). This multiple scattering can be illustrated with Feynman's diagrams. Formally it can be written like that:

$$\tilde{E}_m = \hat{R} * \tilde{E}_{y0}, \qquad (28)$$

where $\hat{R}*$ is the operator summation on all the sequences leading from the falling beam to the m beam. The authors of the discussed works showed, that when plane waves interact, then the suggested calculation method leads to well known results (the formulas (10) and (14)). This method was also used by the authors to describe the acoustooptical interaction in the near-Bragg's range, where other analytic methods are not enough accurate (comp. [33, 34]). It must be added, that in a general case the method is very arduous and computers of a high calculation power should be used.

A very wide Fourier's analysis of interactions of acoustooptical beams of arbitrary amplitudes distribution was done in the book[4]. The equation (3) the authors write like that:

$$\Delta E - \frac{n^2}{c^2}\frac{\partial^2 E}{\partial t^2} = \frac{2n\Delta n}{c^2}\frac{\partial^2}{\partial t^2}\left(\mathrm{a}(x, y, z, t)E\right), \qquad (29)$$

where a(x,y,z,f) is a normalized to unity function, which describes space-time changes of the refraction index. These changes were written with the help of the Fourier's expansion

$$\mathrm{a}(x, y, z, t) = \frac{1}{8\pi^3}\int\int_{-\infty}^{+\infty}\int\tilde{\mathrm{a}}(z, q_x, q_y, \Omega)\exp\left[i(\vec{q}\circ\vec{r} - \Omega t)\right]dq_x dq_y d\Omega. \qquad (30)$$

It must be noticed, that the refraction index changes describe relations (5) and (6), which include the acoustic field gradient and the brought function a(x,y,z,f) must express itself by that gradient. Another important equation is the formula of the light wave electric field Fourier's expansion in the interaction region, which has the shape:

$$E(x, y, z, t) = \frac{1}{8\pi^3}\int\int_{-\infty}^{+\infty}\int\left[\tilde{E}^{+}(z, k_x, k_y, \omega)e^{ik_z z} + \tilde{E}^{-}(z, k_x, k_y, \omega)e^{-ik_z z}\right]\exp\left[i(k_x x + k_y y - \omega t)\right]dk_x dk_y d\omega, \qquad (31)$$

but the "gauge" condition should be fulfilled:

$$\frac{\partial\tilde{E}^{+}}{\partial z}e^{ik_z z} + \frac{\partial\tilde{E}^{-}}{\partial z}e^{-ik_z z} = 0. \qquad (32)$$

The $\tilde{E}^{+}$ function describes the waves which run in the plus direction of the OZ axis, and the $\tilde{E}^{-}$ function describes the waves which run in the opposite direction, so the waves which are reflected from the region with the acoustic wave and/or other optical elements which are behind that region in the direction of the falling wave propagation. In that case the falling wave can be described by the formula:

$$E_y = \frac{1}{8\pi^3}\int\int\int_{-\infty}^{+\infty} \tilde{E}_y^{+}(z_l,k_x,k_y,\omega)\exp[i(k_x x + k_y y - \omega t)]dk_x dk_y d\omega . \tag{33}$$

When we put the dependences (30) and (31) into (29), a system of equations for the searched $\tilde{E}^{\pm}$ function is obtained. The authors solve this system in various specific situations, but it is not necessary to discuss them here. In particular they describe when it is necessary to take into consideration the reflected waves. It is necessary, among others, for the enough narrow interaction region in Raman-Nath interaction and also in the Bragg's interaction when the wave vector of the acoustic wave has an enough big component in the direction of light wave propagation.

In closing this short review of the works devoted to Fourier transform method application in acoustooptic it must be said that it is really incomplete. However, the purpose was to present the idea of the method itself, its possibilities, the used assumptions and applied simplifications. Among the missed works, the author thinks that works of Glinski[35, 36, 37], Henderson[38], other works of Korpel and co[39, 40, 41, 42], Vanaverbeke and Loroy's work[43], and – one of the latest – Parygin's and co[44, 45, 46] works should be mentioned.

4.2. Ray (geometrical optics) methods

The geometrical optics methods in the acoustooptical effect are in fact in two ways applied. Firstly, as auxiliary methods which relatively easy allow to determine some of the qualitative geometrical aspects of that effect. Next, the results can be the base to form adequate (simplified) wave equations, or also integral ones. The work of Fox[47] and the works of Korpel and co[39, 48] should be counted into that kind of works. A wide report on the possibilities of such description in one of Korpel's work[49] can be found. The works, in which the eikonal theory and Debye's (Luneburg-Klein's)[9] expansion is consistently used, belong to the second group. That method is completely equivalent to the solution of the Helmholtz equation for the given wave equation (for example (3)). The works [12, 13] belong to this group.

As it has already been mentioned, the first method in which geometrical optics methods to describe the acoustooptical effect was used, were the work of Lucas[6]. Next works were published by Nomoto[50] and Pouliquen and Segard[51]. If the Lucas's work is often quoted in literature (as one of the first works about acoustooptics), other of the mentioned works are not noticed. In all these works it was proved, that light rays in the elastic wave field get deflected. It's an expected result because the elastic wave – through the photoelastic effect (5) – generates gradients of the medium refraction index. A generally known fact resulting from geometrical optics rules (comp. [3, 9]) is the light rays deflection in the direction of higher refractive index values. That effect is used in light refractive acoustooptical deflectors. That fact contradicts an assumption that a acoustic diffractive net is a purely phase net. Anyway, that assumption can be enough accurately fulfilled only under precise conditions: not too high refractive index gradients (low frequencies of the acoustic wave) and/or not too long interaction ways. As it is known, these conditions are fulfilled in Raman-Nath interaction. An adequate version of eikonal theory for Bragg's interaction was by Korpel[52] worked out. In these cases the results obtained from geometrical optics methods do not differ from the ones obtained with wave methods and that is why they will not be here in detail discussed.

4. GAUSSIAN OPTICAL BEAM STRONG RAMAN-NATH INTERACTION – COMPLEX GEOMETRICAL OPTICS APPROACH

Formally the geometrical optics methods can be used in arbitrary field distributions in a light wave. The usability of various distributions depends on whether they describe really existing beam, but also, to a large extend, on their complexity. In contemporary optics very often laser beams of gaussian light intensity distribution are used. A very effective method within the confines of complex geometrical optics was worked out to describe such beams[10, 53].

4.3. Gaussian beam in homogeneous medium

In a scalar paraxial approximation the gaussian beam amplitude distribution can be written in the following form[54, 55]:

$$E_g(\vec{r}) = E_0 \frac{z_R}{z_{RL} + iz}\exp\left[ik_0 n_0\left((z-L) + i\frac{x^2+y^2}{2(z_{RL}+iz)}\right)\right]. \tag{34}$$

The plane $z = 0$ is the input plane of that beam, which propagates along the OZ axis (the beam axis is at $x = y = 0$) and its waist is at $z = L$. The quantities

$$z_{RL} = z_R - iL\ , \quad z_R = k_0 n_0 a^2\ , \tag{35}$$

where z_R is called the Rayleigh length. In this situation z_{RL} can be called complex Rayleigh length.

For that beam rays equations are: i) for the ray "momentum":

$$\vec{p} = \frac{d\vec{r}}{d\tau} = \vec{p}_0 = [p_{0x}, p_{0y}, p_{0z}] = \left[\frac{in_0\xi}{z_{RL}}, \frac{in_0\eta}{z_{RL}}, n_0\sqrt{1+\frac{\xi^2+\eta^2}{z_{RL}^2}}\right] = const\ , \tag{36}$$

ii) for the ray coordinates

$$\vec{r}_0 = [\xi, \eta, 0]\ , \quad \vec{r} = \vec{r}_0 + \vec{p}_0\tau\ ,$$
$$x(\tau) = \xi + \frac{in_0\xi}{z_{RL}}\tau, \quad y(\tau) = \eta + \frac{in_0\eta}{z_{RL}}\tau, \quad z(\tau) = \tau n_0\sqrt{1+\left(\xi^2+\eta^2\right)/z_{RL}^2}\ . \tag{37}$$

Here (ξ, η) are coordinates of the ray starting point in the plane $z = 0$ (in the presented situation they coincide with the Cartesian (x, y) coordinates on that plane), and τ is the running coordinate along the ray.

It demands an emphasis that the geometrical optics description of the gaussian beam is fully equivalent to that one obtained by the wave equation solution in near-axis approximation. This expression has a simple physical interpretation[56] – it presents the spherical wave with its source putted in the complex point. Rays (presented by (36)) runs in 6-dimensional complex space and become visible in the real 3-dimensional space when all imaginary parts of their coordinates vanish. It means that to find the gaussian beam amplitude in the given detecting point (x_D, y_D, z_D) we have to solve so called reversal problem of the geometrical optics, i.e. extract variables ξ, η and τ from the equation (37). After linearization this solution have the form:

$$\tau = \frac{z_D}{n_0\sqrt{1+\left(\xi^2+\eta^2\right)/z_{RL}^2}} \cong \frac{z_D}{n_0}\left(1-\frac{\xi^2+\eta^2}{2z_{RL}^2}\right) \cong \frac{z_D}{n_0}\ , \quad \xi = x_D\left(1+\frac{in_0\tau}{z_{RL}}\right)^{-1} \cong x_D\left(1+\frac{iz_D}{z_{RL}}\right)^{-1}, \quad \eta \cong y_D\left(1+\frac{iz_D}{z_{RL}}\right)^{-1}. \tag{38}$$

4.4. Gaussian beam in medium disturbed by an ultrasound beam

In the presence of the ultrasound wave the dielectric constant distribution in the medium has the shape:

$$n^2(\vec{r},t) = n_0^2 + \nu(\vec{r},t),$$
$$\nu(\vec{r},t) = 2n_0\Delta n_0 \exp\left(-\alpha(x+h)\right)\cos\left(q(x+h) - \Omega t + \Phi\right)\Delta H(z_l, z_p)\ , \quad \Delta H(z_l,z_p) = H(z-z_l) - H(z-z_p)\ . \tag{39}$$

In (39) α is the acoustic wave attenuation coefficient and $x = -h$ is the position of the ultrasound transducer (or acoustic wave input plane). Moreover, $H(z)$ denotes Heaviside step function. It means that the damped ultrasound wave with angular frequency Ω causes the small changes with amplitude $\Delta n_0 \ll n_0$ (on transducer surface) in the refraction index of the medium. There the refraction index changes in the OY direction are neglected. The sound propagates in column sharply bounded at $z = z_l$ and $z = z_p$.

4.4.1. Corrections to rays trajectories

Because $|\nu(\vec{r},t)| \ll n^2(\vec{r})$, the first correction to the ray trajectory in the homogeneous medium is given by[14, 11]

$$\vec{r}_1 = \int_0^\tau (\tau - \tau')\frac{1}{2}\vec{\nabla}\nu(\vec{r}_0(\tau'))d\tau' = [x_1(\xi,\tau), 0, z_1(\xi,\tau)], \tag{40}$$

where integration is along undisturbed ray (35). Using (39) we obtain:

$$x_1(\xi,\tau) \equiv \Delta n_0 n_0(\tau - \tau_s)N_x(\xi,\tau_s)\tau_{pl}\ ,$$
$$z_1(\xi,\tau) = \begin{cases} 0 & \tau \le \tau_l \\ \Delta n_0(\tau-\tau_l)N_{zl}(\xi) & \tau_l < \tau \le \tau_p\ , \\ \Delta n_0\left[(\tau-\tau_l)N_{zl}(\xi) - (\tau-\tau_p)N_{zp}(\xi)\right] & \tau > \tau_p \end{cases} \tag{41}$$

where:

$$N_x(\xi,\tau_s) = -\sqrt{\alpha^2+k_a^2}\exp\left[-\alpha\left(x_0(\xi,\tau_s)+h\right)\right]\sin\left[q\left(x_0(\xi,\tau_s)+h\right)-\Omega t+\Phi+\varphi_\alpha\right],$$
$$N_{zl}(\xi) = \exp\left[-\alpha\left(x_0(\xi,\tau_l)+h\right)\right]\cos\left[q\left(x_0(\xi,\tau_l)+h\right)-\Omega t+\Phi\right],$$
$$N_{zp}(\xi) = \exp\left[-\alpha\left(x_0(\xi,\tau_p)+h\right)\right]\cos\left[q\left(x_0(\xi,\tau_p)+h\right)-\Omega t+\Phi\right], \tag{42}$$

and

$$\tau_s = \frac{1}{2}\left[(\tau+\tau_l)H(\tau-\tau_l)+(\tau_p-\tau)H(\tau-\tau_p)\right], \quad \tau_{pl} = (\tau-\tau_l)H(\tau-\tau_l)-(\tau-\tau_p)H(\tau-\tau_p),$$
$$x_0(\xi,\tau_r) = \xi\left(1+i\frac{n_0\tau_r}{z_{RL}}\right) \text{ for } r=s,l,p, \quad \tau_l=\tau(z_l)\cong\frac{z_l}{n_0}=\tau_{l0}, \quad \tau_p=\tau(z_p)\cong\frac{z_p}{n_0}=\tau_{p0}. \tag{43}$$

Additionally $\mathrm{tg}(\varphi_\alpha) = (\alpha/k_a)$. In integration for $x_1(\xi,\tau)$ the approximate integration by the middle point method was used. Now, for $\tau > \tau_p$ (standard region for detection) corrected ray trajectory equations are:

$$\begin{cases} x(\xi,\tau) = x_0(\xi,\tau)+x_1(\xi,\tau) = \xi\left(1+i\dfrac{n_0\tau}{z_{RL}}\right)+\Delta n_0 n_0(\tau-\tau_s)(\tau_p-\tau_l)N_x(\xi,\tau_s) \\ y(\eta,\tau) = y_0(\eta,\tau) = \eta\left(1+i\dfrac{n_0\tau}{z_{RL}}\right) \\ z(\xi,\eta,\tau) = z_0(\xi,\eta,\tau)+z_1(\xi,\tau) = \tau p_{0z}+\Delta n_0\left[(\tau-\tau_l)N_{zl}(\xi)-(\tau-\tau_p)N_{zp}(\xi)\right] \end{cases}, \tag{44}$$

where $\tau_s = (\tau_l+\tau_p)/2$.

4.4.2. Reversal problem solution

For further calculation the reversal problem solution for the detection point (x_D, y_D, $z_D > z_p$) is needed (comp. (38)). Unfortunately, equations (44) are nonlinear. In first order of accuracy relatively to Δn_0 and paraxial corrections results from (44)

$$\tau_l \cong \tau_{l0}\left(1-\frac{\xi_D^2+\eta_D^2}{2z_{RL}^2}\right), \quad \tau_p \cong \tau_{p0}\left(1-\frac{\xi_D^2+\eta_D^2}{2z_{RL}^2}-\frac{\Delta n_0}{n_0}\frac{\Delta z}{z_p}N_{zl}(\xi_D)\right),$$
$$\tau_s = \frac{\tau_l+\tau_p}{2} \cong \tau_{s0}\left(1-\frac{\xi_D^2+\eta_D^2}{2z_{RL}^2}-\frac{\Delta n_0}{n_0}\frac{\Delta z}{2z_s}N_{zl}(\xi_D)\right), \quad \tau_{s0} = \frac{\tau_{l0}+\tau_{p0}}{2} = \frac{z_l+z_p}{2n_0} = \frac{z_s}{n_0},$$
$$\tau_{pl} = \tau_p-\tau_l \cong \tau_{pl0}\left(1-\frac{\xi_D^2+\eta_D^2}{2z_{RL}^2}-\frac{\Delta n_0}{n_0}N_{zl}(\xi_D)\right), \quad \tau_{pl0} = \tau_{p0}-\tau_{l0} = \frac{\Delta z}{n_0} \tag{45}$$
$$\tau_D \cong \frac{z_D}{n_0}\left(1-\frac{\xi_D^2+\eta_D^2}{2z_{RL}^2}+\frac{\Delta n_0}{n_0}N_z(\xi_D)\right), \quad N_z(\xi_D) = \left(\frac{z_l}{z_D}-1\right)N_{zl}(\xi_D)-\left(\frac{z_p}{z_D}-1\right)N_{zp}(\xi_D).$$

In all quantities, except for τ_l, corrections caused by acoustic waves can be seen. All theses corrections are caused by refraction of light rays on the boundaries of the acoustic wave column, or more generally by the gradient of refraction index along the light propagation.

For completeness we need expression for η_D and ξ_D. Both these quantities should contain corrections caused by refraction index gradient. For first of them we obtain:

$$\eta_D \cong y_D\left(1+\frac{iz_D}{z_{RL}}\right)^{-1}-i\frac{\Delta n_0}{n_0}\frac{y_D z_D}{z_{RL}}\left(1+\frac{iz_D}{z_{RL}}\right)^{-2}N_z(\xi_D) = \eta_{D0}+\eta_{D1}. \tag{46}$$

The appearance of this correction in the expression for η_D is especially interesting, because there are no refractive index changes in that direction – its source is only the light wave refraction on the edges of the interaction region, which introduces correction to τ_D. For calculation of ξ_D we rewrite the first equation at (45) in the form:

$$\xi_D \cong x_D\left(1+\frac{iz_D}{z_{RL}}\right)^{-1}-\frac{\Delta n_0}{n_0}\left(1+\frac{iz_D}{z_{RL}}\right)^{-1}\left[(z_D-z_s)\Delta z N_x(\xi_D,\tau_{s0})+i\frac{x_D z_D}{z_{RL}}\left(1+\frac{iz_D}{z_{RL}}\right)^{-1}N_z(\xi_D)\right] = \xi_{D0}+f(\xi_D). \tag{47}$$

In this form that equation can be solved by very effective iterative method, which can be depicted for $(n+1)$ order as

$$\xi_D^{(n+1)} = \xi_{D0}+f\left(\xi_D^{(n)}\right), \quad \xi_D^{(0)} = \xi_{D0}. \tag{48}$$

From numerical valuation results that the first order of that iteration process secures enough precision in used perturbation calculus, i.e.

$$\xi_D \cong \xi_D^{(1)} = \xi_{D0}+f\left(\xi_D^{(0)}\right) = \xi_{D0}+\xi_{D1},$$
$$\xi_{D0} = x_D\left(1+\frac{iz_D}{z_{RL}}\right)^{-1}, \quad \xi_{D1} = -\frac{\Delta n_0}{n_0}\left(1+\frac{iz_D}{z_{RL}}\right)^{-1}\left[(z_D-z_s)\Delta z N_x(\xi_{D0},\tau_{s0})+i\frac{x_D z_D}{z_{RL}}\left(1+\frac{iz_D}{z_{RL}}\right)^{-1}N_z(\xi_{D0})\right]. \tag{49}$$

Accordingly to that in expressions for N_{zl} and N_z in (45) we can use ξ_{D0} instead of ξ_D (in first order of approximation). In other parts of the expressions in (45) we need to use full formulae for ξ_D and η_D. Because of it we obtain:

$$\tau_D = \tau_{D0}+\tau_{D1}, \quad \tau_{D0} = \frac{z_D}{n_0}\left(1-\frac{\xi_{D0}^2+\eta_{D0}^2}{2z_{RL}^2}\right),$$
$$\tau_{D1} = \frac{\Delta n_0}{n_0}\frac{z_D}{n_0}\left\{\frac{x_D}{z_{RL}^2}(z_D-z_s)\Delta z\left(1+\frac{iz_D}{z_{RL}}\right)^{-2}N_x(\xi_{D0},\tau_{s0})+\left[1+i\frac{z_D\left(x_D^2+y_D^2\right)}{z_{RL}^3}\left(1+\frac{iz_D}{z_{RL}}\right)^{-3}\right]N_z(\xi_{D0})\right\}. \tag{50}$$

One can see, that we have two types of corrections to the ray trajectories – deflectional one, proportional to N_x, and refractional, proportional to N_z. In the considered first order of accuracy we have simple situation – to the given point of detection only one ray is coming.

4.4.3. Corrected eikonal

The expression for the considered beam eikonal in perturbation calculus used[11] have the form:

$$\psi(\vec{r}_D) = \psi^0(\vec{r}_0)+\int_0^{\tau_D}\hat{\varepsilon}(\vec{r}(\tau'))d\tau' \equiv \psi^0(\vec{r}_0)+\int_0^{\tau_D}\varepsilon(\vec{r}(\tau'))d\tau'+\frac{1}{2}\int_0^{\tau_D}\nu(\vec{r}(\tau'))d\tau' = \psi^0(\vec{r}_0)+\psi_0^0(\vec{r}_D)+\psi_{10}(\vec{r}_D), \tag{51}$$

where ψ^0 is the boundary value of the eikonal. Here integration is along the corrected rays. Because of it we have (comp. (34)):

$$\psi^0(\vec{r}_0) = -n_0L+in_0\frac{\xi_D{}^2+\eta_D^2}{2z_{RL}} \cong -n_0L+in_0\frac{\xi_{D0}{}^2+\eta_{D0}^2}{2z_{RL}}+in_0\frac{\xi_{D0}\xi_{D1}+\eta_{D0}\eta_{D1}}{z_{RL}} = \psi^{00}+\psi_1^0, \tag{52}$$

$$\psi_0^0(\vec{r}_D) = \int_0^{\tau_D}\varepsilon(\vec{r}(\tau'))d\tau' = \int_0^{\tau_D}n_0^2 d\tau' = n_0^2\tau_{D0}+n_0^2\tau_{D1} = \psi_{00}+\psi_{01}, \tag{53}$$

$$\psi_{10}(\vec{r}_D) = \frac{1}{2}\int_0^{\tau_D}\nu(\vec{r}(\tau'))d\tau' \cong \Delta n_0\Delta z N_\nu(\xi_{D0},\tau_{s0}),$$
$$N_\nu(\xi_{D0},\tau_{s0}) = \exp\left[-\alpha\left(x_0(\xi_{D0},\tau_{s0})+h\right)\right]\cos\left[q\left(x_0(\xi_{D0},\tau_{s0})+h\right)-\Omega t+\Phi\right]. \tag{54}$$

In the last integral the simplified calculation by middle point method was used (like in x_1 calculation (41), (42)). Finally we can divide the eikonal to the undisturbed part ψ_0 and correction ψ_1:

$$\psi_0(\vec{r}_D) = \psi^{00}+\psi_{00} = n_0(z_D-L)+in_0\frac{x_D^2+y_D^2}{2z_{RL}}\left(1+i\frac{z_D}{z_{RL}}\right)^{-1}, \tag{55}$$

$$\psi_1(\vec{r}_D) = \psi_1^0+\psi_{01}+\psi_{10} =$$
$$\Delta n_0\left\{\Delta z N_\nu(\xi_{D0},\tau_{s0})-i\frac{x_D}{z_{RL}}(z_D-z_s)\Delta z N_x(\xi_{D0},\tau_{s0})+\left[1+\left(1+i\frac{z_D}{z_{RL}}\right)^{-2}\frac{x_D^2+y_D^2}{z_{RL}^2}\right]z_D N_z(\xi_{D0})\right\}. \tag{56}$$

One can see that the correction to the eikonal contains three parts: phasial (proportional to N_v), deflectional (proportional to N_x) and refractional (proportional to N_z). All of these corrections are harmonically modulated with acoustics waves frequency. For further calculations we introduce here some shorter notations:

$$\psi_1(\vec{r}_D) = e_{ph}N_v + e_{df}N_x + e_{rf}N_z . \qquad (57)$$

4.4.4. Corrected amplitude

When divergent beam propagates its amplitude changes accordingly with expression[14]:

$$A(\tau) = A(0)\sqrt{\frac{D(0)}{D(\tau)}}, \quad D = \frac{\partial(x,y,z)}{\partial(\xi,\eta,\tau)}, \quad A(0) = E_0\frac{z_R}{z_{RL}} . \qquad (58)$$

Here D is the jacobian for the transition from Cartesian to ray coordinates. The relations between these sets of coordinates are given by (44). In first order of accuracy we have

$$D(\tau) \cong \frac{\partial x}{\partial \xi}\frac{\partial y}{\partial \eta}\frac{\partial z}{\partial \tau} = \left[\left(1+i\frac{n_0\tau}{z_{RL}}\right) + \Delta n_0 n_0 \tau_{pl}(\tau-\tau_s)\frac{\partial N_x(\xi,\tau)}{\partial \xi}\right]\left(1+i\frac{n_0\tau}{z_{RL}}\right)\left[p_{0z} + \Delta n_0\left(N_{zl}(\xi) - N_{zp}(\xi)\right)\right]. \qquad (59)$$

Finally

$$A(\vec{r}_D) = A_0(\vec{r}_D)(1+a_1(\vec{r}_D)), \quad A_0(\vec{r}_D) = E_{y0}\frac{z_D}{z_{RL}}\left(1+i\frac{z_D}{z_{RL}}\right)^{-1}$$

$$a_1(\vec{r}_D) = \frac{\Delta n_0}{n_0}\left\{-\frac{1}{2}\Delta z(z-z_s)\left(1+i\frac{z_D}{z_{RL}}\right)^{-1}\frac{\partial N_x(\xi_{D0},\tau_{s0})}{\partial \xi_{D0}} - i\frac{x_D z_D}{z_{RL}^3}\Delta z(z-z_s)\left(1+i\frac{z_D}{z_{RL}}\right)^{-3} N_x(\xi_{D0},\tau_{s0}) + \right.$$

$$\left. -i\frac{z_D}{z_{RL}}\left(1+i\frac{z_D}{z_{RL}}\right)^{-1}\left[1+iz_D\frac{x_D^2+y_D^2}{z_{RL}^3}\left(1+i\frac{z_D}{z_{RL}}\right)^{-3}\right]N_z(\xi_{D0})\right\} = a_{gr}\frac{\partial N_x}{\partial \xi_{D0}} + a_{df}N_x + a_{rf}N_z . \qquad (60)$$

Similarly to the expression for eikonal (56) this expression for light beam amplitude correction contains three different parts. All of these parts influence divergence of that beam. The first part can be called gradiental, the second one deflectional and the last one refractional, respectively to the physical reasons causing its creation. In the last line in (60) next shortened notations are introduced. It is clear that all parts of this correction are harmonically modulated with acoustics waves frequency.

4.4.5. Disturbed gaussian beam electric field distribution

Finally, we can write the expression for electric field distribution in gaussian beam modified by ultrasound in considered interaction:

$$E_y(\vec{r}_D) \cong A_0(\vec{r}_D)[1+a_1(\vec{r}_D)]\exp[ik(\psi_0(\vec{r}_D)+\psi_1(\vec{r}_D))] = E_g(\vec{r}_D)[1+a_1(\vec{r}_D)]\exp[ik(\psi_1(\vec{r}_D))], \qquad (61)$$

where E_g is undisturbed gaussian beam distribution given by (34). Moreover, taking into account that both quantities a_1 and ψ_1 are complex, in first order of accuracy we have:

$$1+a_1(\vec{r}_D) = 1+a_{1R}(\vec{r}_D)+ia_{1I}(\vec{r}_D) \cong (1+a_{1R}(\vec{r}_D))\exp(ia_{1I}(\vec{r}_D)),$$

$$\exp[ik(\psi_1(\vec{r}_D))] = \exp[ik(\psi_{1R}(\vec{r}_D)+i\psi_{1I}(\vec{r}_D))] \cong \exp[ik(\psi_{1R}(\vec{r}_D))][1-k\psi_{1I}(\vec{r}_D)]. \qquad (62)$$

In this situation we have

$$E_y(\vec{r}_D) \cong E_g(\vec{r}_D)[1+a_{1R}(\vec{r}_D)-k\psi_{1I}(\vec{r}_D)]\exp[i(k\psi_{1R}(\vec{r}_D)+a_{1I}(\vec{r}_D))]. \qquad (63)$$

After elementary but extensive calculation, because of the harmonic modulation of quantities a_1 and ψ_1, we can write:

$$\psi_1(\vec{r}_D) = [e_{cR}(\vec{r}_D)+ie_{cI}(\vec{r}_D)]\cos(\varphi_s(t)) + [e_{sR}(\vec{r}_D)+ie_{sI}(\vec{r}_D)]\sin(\varphi_s(t)),$$

$$a_1(\vec{r}_D) = [a_{cR}(\vec{r}_D)+ia_{cI}(\vec{r}_D)]\cos(\varphi_s(t)) + [a_{sR}(\vec{r}_D)+ia_{sI}(\vec{r}_D)]\sin(\varphi_s(t)), \qquad (64)$$

where

$$\varphi_s(t) = q[x_{0sR}(\xi_{D0},\tau_{s0})+h] - \Omega t + \Phi, \quad x_{0sR}(\xi_{D0},\tau_{s0}) = \mathrm{Re}\left[\frac{z_R+i(z_s-L)}{z_R+i(z_D-L)}\right]. \qquad (65)$$

Moreover we can write

$$k\psi_{1R}(\vec{r}_D)+a_{1I}(\vec{r}_D) = a_e(\vec{r}_D)\sin(\varphi_s(t)+\varphi_e), \quad a_{1R}(\vec{r}_D)-k\psi_{1I}(\vec{r}_D) = a_f(\vec{r}_D)\sin(\varphi_s(t)+\varphi_f). \qquad (66)$$

Substituting these relations to (63) we obtain:

$$E_y(\vec{r}_D) \cong E_g(\vec{r}_D)[1+a_f(\vec{r}_D)\sin(\varphi_s(t)+\varphi_f)]\exp[ia_e(\vec{r}_D)\sin(\varphi_s(t)+\varphi_e)] =$$

$$E_g(\vec{r}_D)[1+a_f(\vec{r}_D)\sin(\varphi_s(t)+\varphi_f)]\sum_{m=-\infty}^{+\infty} i^m J_m(a_e(\vec{r}_D))\exp[im(\varphi_s(t)+\varphi_e)], \qquad (67)$$

where J_m denote the first type Bessel function of the order m. Physically this extension denotes that the light beam interacting with acoustic wave decomposes on an unlimited quantity of the diffracted beams. Moreover, all diffracted orders have the shifted angular frequency of the value $\Delta\omega_m = m\Omega$ and are modulated with acoustic frequency. It is the extension of standard Raman-Nath solution that uses only phasial correction. Among others mentioned above corrections, the acoustic wave attenuation is also taken into account.

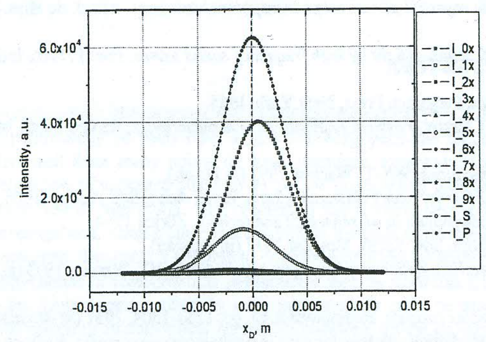

Fig. 2. Light intensity distribution in diffractive orders for "classical" Raman-Nath interaction when only phasial correction is taken into account. I_0x÷I_9x correspond to $I_0(x_D)\div I_9(x_D)$. I_P denotes the incident light intensity distribution, I_S is the sum of intensities in all calculated orders. For used data (see text) two latter curves are nearly identical. An asymmetry caused by acoustic wave attenuation is visible.

Considering that the acoustic wave frequency is much smaller then the light wave frequency, we can calculate the diffracted beams intensities as

$$I_m(\vec{r}_D) \cong I_g(\vec{r}_D)[1+2a_f(\vec{r}_D)\sin(\varphi_s(t)+\varphi_f)]J_m^2(a_e(\vec{r}_D))' \qquad (68)$$

where I_g is the light intensity distribution in undisturbed gaussian beam. In acoustooptics, in many situations diffraction orders intensity modulation is not important – because of its high frequency (many MHz) phase sensitivity detectors (lock-in amplifiers) for its registration do not exist. If the detector measures the mean light intensity (in time) and is equipped with the pin-hole at $y_D = 0$ then we obtain

$$I_m(x_D) \cong I_g(x_D)J_m^2(a_e(x_D)). \qquad (69)$$

Using the expression (69) we can analyze the effect of different experimental parameters on the light intensity distribution in diffracted beams.

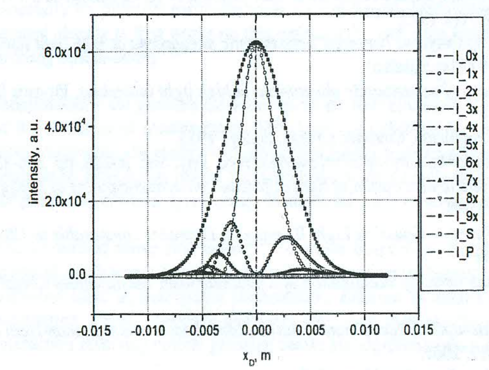

Fig. 3. Light intensity distribution in diffractive orders in considered Raman-Nath interaction when all corrections are taken into account. All data are the same as in Fig. 2. The "fine structure" in diffraction orders is visible. The visible asymmetry is caused by acoustic wave attenuation, like in "classical" case.

For the illustration of above theoretical considerations there are two graphs for final formula presented below. The calculations were performed for next values of parameters: $\lambda = 636$ nm, $a = 4$ mm, $L = 0{,}5$ m, $h = 1$ cm, $n_0 = 1{,}5$, $f_a = 3$ MHz (acoustic wave frequency), $\upsilon_a = 1500$ m/s (acoustic wave velocity), $\alpha = 75$ 1/m, $\Delta n_0 = 2\cdot 10^{-4}$, $z_l = 0{,}6$ m, $\Delta z = 1$ mm, $z_D = 0{,}7$ m. For these data we have $p_{RN} = 1{,}98$ and $Q = 0{,}025$ (Klein-Cook parameter). Moreover $2a \approx 20\lambda_a$ so there are good conditions for Raman-Nath light diffraction.

There are evidently important differences between Fig. 2 and Fig. 3. There is observed a very interesting fine structure in diffraction orders light intensity distribution. It can be seen that for some positions x_D light intensity vanish. It is possible when the appropriate Bessel function reaches its zeros. It results from numerical valuation that in most situations the main reason for these differences is light ray deflection in refraction index gradient field. But for other data the other corrections considered above can give important contribution to the light intensity distribution in diffractive orders. Especially it concerns refractional correction that appears in both eikonal and amplitude of considered gaussian beam interacted with acoustic column (similarly like deflectional correction). It should be noticed that this correction was not taken into account in other theories of Raman-Nath interaction.

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