1998 rok   Załącznik nr 16

Complex geometrical optics application to the gaussian light beam Raman-Nath diffraction description

R. J. Bukowski
Institute of Physics, Silesian Technical University, ul. Krzywoustego 2, 44-100 Gliwice, Poland

ABSTRACT

The description of the gaussian light beam diffraction on the planar ultrasound wave in Raman-Nath region by complex optical rays method is presented in this works. The case of the perpendicular light beam incidence (i.e. its axis) relatively to the direction of propagation of the ultrasound beam of Δz width was considered. The influence of such parameters as a size and position of the gaussian beam waist, a laser-screen (detector) distance, a position of the ultrasound beam and Raman-Nath parameter value on diffraction pattern was analysed. That was proved that in some cases the diffraction beams have a fine structure. The final analytical formulas were illustrated by some graphs of the light intensity distributions in beams diffracted on ultrasound waves propagated in water.

Keywords: Raman-Nath diffraction, complex geometrical optics, gaussian light beams, geometrical optics of nonhomogeneous media, perturbation calculus.

1. INTRODUCTION

Light beams with the gaussian profile of the light intensity distribution are widely applied in scientific and commercial devices. It is caused by their unique properties, in which are small angular divergence, stability, disturbances missing in many optical systems. Because of that properties the theoretical description of the gaussian beams propagation is relatively simple.

When the gaussian beam is propagated in optically nonhomogeneous medium the situation is more complicated. I that case the intensity distribution in beam profile can undergo essential change. These changes contain an information, which may be very important. This situation we have, e.g., in acoustooptical interactions or in photothermal investigations with photodeflectional detection (mirage effect).

The full description of the gaussian beam propagation in that conditions need the wave optics law applications, i.e. the appropriate wave equation solution. In many cases the exact solutions of that equation are unknown. In this situation approximative methods become very important. For acoustooptical interactions many effective methods for wave equation approximate solution were worked out, especially for different particular cases, as e.g. Raman-Nath interaction or Bragg interaction. These methods went into acoustooptics theoretical grounds and it may be found in many school-books[1, 2, 3]. It must be marked that these methods are mathematically complicated in general, especially when they are used for the description of wave beams with complex amplitude and phase distributions interaction.

Mentioned above approximative methods for acoustooptics wave equations solutions were applied already in first works on this topic, e.g.[4, 5]. Independently of that at initial stage of acoustooptics evolution the geometrical optics methods were used too, e.g. in one of the first work[6]. That work and another ones similar to that are discussed in books[7, 8]. Because of wave theory successes the geometrical optics methods were forgotten for many years. But in last years an important progress in geometrical optics was attained. Thanks to that these methods become competitive relatively to "exact" wave optics methods, especially for problems, in which approximative methods for wave equation solution are used (because of different reasons).

In present work, geometrical optics method for Raman-Nath acoustooptical interaction description was used. In first part of that work the geometrical optics fundamental equations[9, 10, 11] for homogeneous and nonhomogeneous media are discussed. Next, these equations are used for gaussian beam propagation description in medium disturbed by the plane ultrasound wave for the case when both waves run perpendicularly each other. Obtained results are illustrated graphically for the case when considered acoustooptical interaction happens in water.

2. GEOMETRICAL OPTICS EQUATIONS

The information about light intensity distribution in monochromatic beam with angular frequency ω propagated in given medium in the Helmholtz equation is contained (with appropriate boundary conditions):

$$\Delta u(\vec{r}) + k_0^2 \varepsilon(\vec{r}) u(\vec{r}) = 0, \tag{1}$$

where

$$k_0 = \frac{\omega}{c}, \quad \varepsilon(\vec{r}) = n^2(\vec{r}). \tag{2}$$

In these equations are used followed marks: c = light velocity in space; $\varepsilon = n^2$ dielectric constant and refraction coefficient of the medium. The fundamental solutions of the equation (1) for the homogeneous medium with refraction coefficient n_0 are very well known. For example, the solution in the plane wave form may be written as

$$u(\vec{r}) = Ae^{i\Psi(\vec{r})}, \quad \Psi(\vec{r}) = k_0 n_0 \vec{r} \circ \vec{e}_k, \tag{3}$$

where $\vec{e}_k$ denote an unit vector in the wave propagation direction.

In general case (any wave, any medium) Eqn. (1) solutions are looked for in the shape

$$u(\vec{r}) = A(\vec{r}) e^{i\Psi(\vec{r})}, \quad \Psi(\vec{r}) = k_0 \psi(\vec{r}), \tag{4}$$

where

$$A(\vec{r}) = \sum_{m=0}^{\infty} \frac{A_m(\vec{r})}{(ik_0)^m}. \tag{5}$$

The above wave amplitude expansion on "partial amplitudes" is called Debye expansion or Luneburg-Klein expansion. The series (5) convergence is the faster as the wave number k_0 is bigger. After assumed solution substitution to Helmholtz eqn. (1) we obtain the set of differential equations for A_m amplitudes:

$$\begin{cases} (\vec{\nabla}\psi)^2 = n^2 \\ 2(\vec{\nabla}A_0) \circ (\vec{\nabla}\psi) + A_0 \Delta\psi = 0 \\ 2(\vec{\nabla}A_0) \circ (\vec{\nabla}\psi) + A_1 \Delta\psi = -\Delta A_0 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ 2(\vec{\nabla}A_0) \circ (\vec{\nabla}\psi) + A_m \Delta\psi = -\Delta A_{m-1} \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \end{cases} \tag{6}$$

The first equation from that set is called as eikonal equation, and other ones are called as transport equation of zero, first and so on order.

The boundary conditions for Helmholtz equation have a shape of wave amplitude value $u = u^0(\xi, \eta)$ distribution given on some surface Q defined e.g. by the parametric equation

$$\vec{r} = \vec{r}^{\,0}(\xi, \eta), \tag{7}$$

where ξ and η denoted the coordinates on the surface Q (generally curvilinear). In order to transfer that conditions on the geometrical optics ground the function $u^0(\xi, \eta)$ must be expanded on Debye series:

$$u^0(\xi,\eta) = \sum_{m=0}^{\infty} \frac{A_m^0(\xi,\eta)}{(ik_0)^m} \exp\left(ik_0 \psi^0(\xi,\eta)\right). \tag{8}$$

From that:

$$\psi\big|_Q = \psi^0(\xi,\eta), \quad A_m\big|_Q = A_m^0(\xi,\eta). \tag{9}$$

2.1. General solution of the geometrical optics equations

The eikonal equation solution with initial condition (9) can be written as

$$\psi(\vec{r}) = \psi^0(\xi,\eta) + \int_0^\tau n^2\left[\vec{r}(\tau')\right]d\tau', \tag{10}$$

where integration is carried out along the characteristics curve of the eikonal equation which is the ray trajectory. The variable τ is the current variable along the ray. The equations for the ray can be written in hamiltonian shape

$$\frac{d\vec{r}}{d\tau} = \vec{p}, \quad \frac{d\vec{p}}{d\tau} = \frac{1}{2}\vec{\nabla}n^2(\vec{r}), \tag{11}$$

where

$$\vec{p} = \vec{\nabla}\psi, \quad \frac{d\psi}{d\tau} = \vec{p}^{\,2} = n^2(\vec{r}). \tag{12}$$

From last equations arise the initial conditions for the rays:

$$\vec{p}^{\,0}\big|_{Q} = \vec{\nabla}\psi^0(\vec{r}^{\,0}(\xi,\eta)), \tag{13}$$

from two components of the initial ray "momentum" started from point $\vec{r}^{\,0}$ are calculated. The third component is calculated from the second equation of the set (12) (i.e. eikonal equation):

$$(\vec{p}^{\,0})^2 = n^2(\vec{r}^{\,0}). \tag{14}$$

In order to completing of above solutions we must calculate yet the change of the partial amplitudes along the ray. The appropriate expression for zero order amplitude has a shape

$$A_0(\tau) = A_0(0)\left[\frac{D(0)}{D(\tau)}\right]^{1/2}, \tag{15}$$

where

$$D(\tau) = \frac{\partial(x,y,z)}{\partial(\xi,\eta,\tau)}, \tag{16}$$

is the jakobian for the transition from cartesian coordinates (x, y, z) to curvilinear coordinates (ξ, η, τ). The solutions for the higher order transport equations are known too, but in this work they are not explored.

2.2. Perturbation calculus for ray equation

How can be seen from the previous section the general solutions of the geometrical optics equations are well known, but the appropriate integral calculations in general case may be very difficult or may be lead to very complicated expressions. In this situation the approximative calculations methods are acquired a big importance. When we have

$$n^2(\vec{r}) = n_0^2(\vec{r}) + \nu(\vec{r}) \Leftrightarrow \varepsilon(\vec{r}) = \varepsilon_0(\vec{r}) + \nu(\vec{r}) \tag{17}$$

then eikonal and rays equations take the shape

$$(\vec{\nabla}\psi)^2 = n_0^2(\vec{r}) + \nu(\vec{r}), \quad \frac{d^2\vec{r}}{dt^2} = \frac{1}{2}\vec{\nabla}n_0^2(\vec{r}) + \frac{1}{2}\vec{\nabla}\nu(\vec{r}). \tag{19}$$

When the disturbance is small, i.e.

$$|\nu(\vec{r})| << \varepsilon_0(\vec{r}) \tag{18}$$

then considered solutions are looked for in the shape

$$\vec{r}(\tau) = \vec{r}_0(\tau) + \vec{r}_1(\tau) + \vec{r}_2(\tau) + \ldots, \tag{20}$$

where the ray trajectory in undisturbed medium is given by equation

$$\frac{d^2\vec{r}_0}{d\tau^2} = \frac{1}{2}\vec{\nabla}n_0^2(\vec{r}). \tag{21}$$

For the first order correction we can write

$$\frac{d^2}{d\tau^2}\left(\vec{r}_0+\vec{r}_1\right)=\frac{1}{2}\vec{\nabla}n_0^2\left(\vec{r}_0+\vec{r}_1\right)+\frac{1}{2}\vec{\nabla}\nu\left(\vec{r}_0+\vec{r}_1\right), \tag{22}$$

and after the expansion fulfilment

$$n_0^2\left(\vec{r}_0+\vec{r}_1\right)=n_0^2\left(\vec{r}_0\right)+\left(\vec{\nabla}n_0^2\left(\vec{r}_0\right)\right)\circ\vec{r}_1,\quad \nu\left(\vec{r}_0+\vec{r}_1\right)=\nu\left(\vec{r}_0\right)+\left(\vec{\nabla}\nu\left(\vec{r}_0\right)\right)\circ\vec{r} \tag{23}$$

we obtain the equation for correction to ray trajectory in the shape

$$\frac{d^2\vec{r}_1}{d\tau^2}\cong\frac{1}{2}\left(\vec{r}_1\circ\vec{\nabla}\right)\vec{\nabla}n_0^2\left(\vec{r}_0\right)+\frac{1}{2}\vec{\nabla}\nu\left(\vec{r}_0\right). \tag{24}$$

After the solution of this equation we calculate eikonal:

$$\psi\left(\vec{r}\right)=\psi^0(\xi,\eta)+\int_0^\tau\left[n_0^2\left(\vec{r}_0+\vec{r}_1\right)+\nu\left(\vec{r}_0+\vec{r}_1\right)\right]d\tau'. \tag{25}$$

As it is seen, the integration is carried out along the corrected ray trajectory. In simpler version, when the correction to the ray trajectory can be neglected, the integration along undisturbed ray trajectory may be used. In the next step we calculate wave amplitude by jakobian calculation with appropriate accuracy:

$$A_0(\tau)=A_0(0)\left[\frac{D(0)}{D(\tau)}\right]^{1/2},\quad D(\tau)=\frac{\partial\left(\vec{r}_0+\vec{r}_1+\ldots\right)}{\partial(\xi,\eta,\tau)}. \tag{26}$$

3. DESCRIPTION OF THE GAUSSIAN BEAM PROPAGATION

3.1. Gaussian beam in homogeneous medium

In accordance with the theory given in Sect. 2 the initial condition must be given and adequate equations must be solved. The characteristic place in gaussian beam is the waist, in which the light wave has the plane wave shape and its amplitude distribution by Gauss function is described. Assuming that the waist is in the plane $z = 0$ we can write:

$$u_0(x,y)=E_0\exp\left[-\left(x^2+y^2\right)/2a^2\right], \tag{27}$$

where a denote the gaussian beam radius (for the light intensity), and E_0 denote the electric field intensity in the beam centre. The Debye series for that function is very easy to obtain:

$$u_0(x,y)=E_0\exp\left[ik\left(i\left(x^2+y^2\right)/2a^2k\right],\right. \tag{28}$$

i.e.

$$\psi^0=k\varphi_0,\quad \varphi_0=i\left(x^2+y^2\right)/2ka^2. \tag{29}$$

Assuming that the ray start point coordinates from plane z = 0 are

$$\vec{r}_0=[\xi,\eta,0], \tag{30}$$

(where ξ and η are the cartesian coordinates of that point which are cover its x and y coordinates) we can calculate the initial ray "momentum" components on that plane:

$$\vec{p}_0=\vec{\nabla}\psi^0\Leftrightarrow\left[p_{0x}=\frac{i\xi}{ka^2},\quad p_{0y}=\frac{i\eta}{ka^2}\right]. \tag{31}$$

The third component of that ray "momentum" in the homogeneous medium with refraction coefficient n_0 is calculated from equation (cf. Eqn. (14)):

$$\left(\vec{p}_0\right)^2=p_{0x}^2+p_{0y}^2+p_{0z}^2=n_0^2, \tag{32}$$

i.e.

$$p_{0z}=\pm\sqrt{n_0^2+\frac{\xi^2+\eta^2}{\left(ka^2\right)^2}}. \tag{33}$$

The sign is selected relatively to the direction of beam propagation – in considered case was assumed that the wave propagates in positive direction of the z axis (sing +).

In this situation the Eqs. (11) solutions are

$$\bar{p} = \bar{p}_0 = \text{const} \ , \tag{34}$$

and

$$x(\tau) = \xi + \frac{i\xi}{ka^2}\tau, \quad y(\tau) = \eta + \frac{i\eta}{ka^2}\tau, \quad z(\tau) = \tau\sqrt{n_0^2 + (\xi^2+\eta^2)/(ka^2)^2} \ . \tag{35}$$

The integration of the eikonal Eqn. (10) is very simple. We obtain (cf. Eqn. (28)):

$$\psi(\tau) = \psi^0 + n_0^2\tau, \quad \psi^0 = i\frac{\xi^2+\eta^2}{2ka^2} \ . \tag{36}$$

Regarding that for the near-axis rays we can write next approximations

$$\tau = \frac{z}{\sqrt{n_0^2 + (\xi^2+\eta^2)/(ka^2)}} \cong \frac{z}{n_0}\left(1 - \frac{\xi^2+\eta^2}{2(ka^2)^2 n_0^2}\right) \cong \frac{z}{n_0} \ , \tag{37}$$

$$\xi = x\left(1 + \frac{i\tau}{ka^2}\right)^{-1} \cong x\left(1 + \frac{iz}{ka^2 n_0}\right)^{-1}, \quad \eta \cong y\left(1 + \frac{iz}{ka^2 n_0}\right)^{-1} \tag{38}$$

we obtain the final formula for the eikonal:

$$\psi(\bar{r}) = zn_0 + i\frac{x^2+y^2}{2ka^2}\left(1 + \frac{iz}{ka^2 n_0}\right)^{-1} = \psi_{0R} + i\psi_{0I} \tag{39}$$

(ψ_{0R} and ψ_{0I} denotes the real and imaginary parts of ψ). In order to determine the wave amplitude change along the ray we must calculate the jakobian for transition between (x, y, z) and (ξ, η, τ) variables. In the frame of assumed approximations the essential elements of matrix (16) are

$$D_{x\xi} = 1 + \frac{iz}{ka^2 n_0}, \quad D_{y\eta} = 1 + \frac{iz}{ka^2 n_0}, \quad D_{z\tau} = \sqrt{n_0^2 + \frac{\xi^2+\eta^2}{(ka^2)^2}} \ , \tag{40}$$

and it gives in accordance with Eqn. (15)

$$A_0 = \left(1 + \frac{iz}{ka^2 n_0}\right)^{-1} \ . \tag{41}$$

In this situation in zero order approximation on the base of Debye series we obtain

$$u(x,y,z) = E_0\left(1 + \frac{iz}{ka^2 n_0}\right)^{-1} \exp\left[ikn_0 z - \frac{x^2+y^2}{2a^2}\left(1 + \frac{iz}{ka^2 n_0}\right)^{-1}\right] . \tag{42}$$

That expression describes the wave amplitude distribution in the gaussian beam obtained in the geometrical optics frame. It needs emphasis that this expression is fully equivalent to that one obtained by the wave equation solution in near-axis approximation (cf. for example[12]). Obtained expression has a simple physical interpretation[13] – it is presented the spherical wave with its source putted in complex point. This expression is in good agreement with that one from the work[14]. At the end it needs noticing, that expression (46) can be easy generalised for the case when gaussian beam started from the plane $z = 0$ and have their waist in the plane $z = L$ – it needs only substitution $z \rightarrow (z - L)$.

3.2. Gaussian beam in homogeneous medium disturbed by ultrasound wave

Let's consider the experimental setup presented at Fig. 1. It is a schematic diagram of the standard setup for the Raman-Nath diffraction investigation. The plane, longitudinal and damped ultrasound wave is propagated in the medium, and the light beam is incident on it at the right angle. The gaussian light beam is emitted by laser and is formed by adequate optical system. The acoustooptical interaction result is observed on the screen or measured by an appropriate detector, e.g. photomultiplier.

The damped ultrasound wave with angular frequency Ω is caused in the medium the small harmonics changes with amplitude Δn in refraction coefficient, and respectively to that the dielectric constant changes can be written in the shape[2]

$$\nu(\bar{r},t) = 2n_0\Delta n \exp(-\alpha(x+h))\cos(k_a(x+h) - \Omega t + \Phi), \tag{43}$$

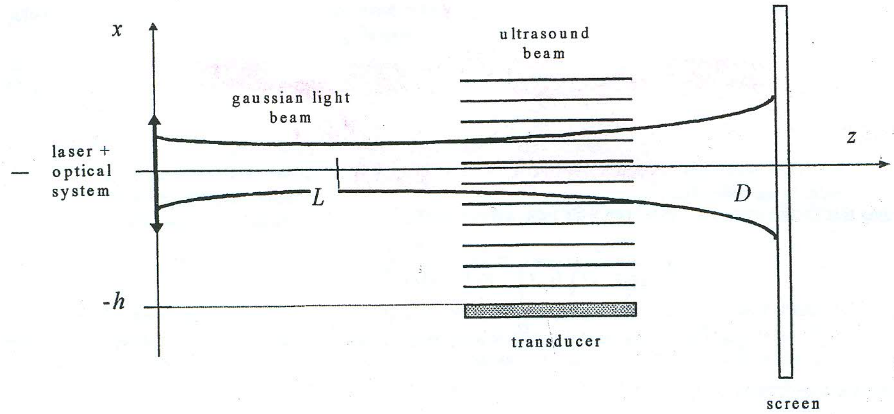

Figure 1. The experimental setup scheme for gaussian light beam Raman-Nath diffraction on plane ultrasound wave investigation. The ultrasound beam has the wide Δz and its left edge is at distance z_l from the reference system centre (putted in the light beam "input"). We assume that the ultrasound width along the OY axis is many times greater then the light beam width. The light beam waist have the radius a and is at distance L from their "input". The screen (or the detector) is at distance z_D from the "input".

where α is the acoustic wave attenuation coefficient and k_a its wave number. In Raman-Nath diffraction analysis traditionally is assumed, that acoustic wave is generated the phase diffraction grating. In this situation we can neglect the light rays curving and appropriate integrals in perturbation calculus can be calculated along the undisturbed gaussian beam rays. The first correction to eikonal is given by expression

$$\psi_1(\vec{r}_D) = \int_0^{\tau_D} \nu(\vec{r}(\tau'))d\tau' = \int_{\tau_l}^{\tau_p} \nu(\vec{r}(\tau'))d\tau', \quad \tau_l = z_l/n_0, \quad \tau_l = (z_l + \Delta z)/n_0, \tag{44}$$

where

$$\nu(\vec{r}(\tau)) = \nu(x(\tau)), \quad x(\tau) = x_D\left(1 + \frac{i\tau n_0}{a_C}\right)\Big/\left(1 + \frac{iz_D}{a_C}\right) = x_R(\tau) + ix_I(\tau), \quad a_C = ka^2 n_0. \tag{45}$$

In (44) we have an elementary integral but because of it properties (and very enlargement shape too) we can use the approximative integration by middle point method, which gives

$$\psi_1(\vec{r}_D) \cong \frac{1}{2}(\nu_R(\tau_s) + i\nu_I(\tau_s))\Delta\tau = \psi_{1R} + i\psi_{1I}, \quad \tau_s = \frac{1}{2}(\tau_l + \tau_p), \quad \Delta\tau = \tau_p - \tau_l = \Delta z/n_0, \tag{46}$$

where

$$\begin{aligned} \nu_R(\tau_s) &= \nu_{Ra}\cos(\beta_R + \beta_c), \quad \nu_{Ra} = 2n_0\Delta n \exp(-\alpha(x_R(\tau_s) + h))[\cos^2(\alpha x_I(\tau_s)) + \mathrm{sh}^2(\beta_I)]\Delta\tau, \\ \beta_R &= k_a(x_R(\tau_s) + h) - \Omega t + \Phi, \quad \beta_I = k_a x_I(\tau_s), \quad \mathrm{tg}\,\beta_c = \mathrm{tg}(\alpha x_I(\tau_s))\mathrm{th}(\beta_I) \end{aligned} \tag{47}$$

and

$$\begin{aligned} \nu_R(\tau_s) &= -\nu_{Ia}\sin(\beta_R + \beta_s), \quad \nu_{Ia} = 2n_0\Delta n \exp(-\alpha(x_R(\tau_s) + h))[\sin^2(\alpha x_I(\tau_s)) + \mathrm{sh}^2(\beta_I)]\Delta\tau, \\ \mathrm{tg}\,\beta_s &= \mathrm{tg}(\alpha x_I(\tau_s))\mathrm{cth}(\beta_I). \end{aligned} \tag{48}$$

Moreover, with accordance to accepted approximations we can assume, that the field amplitude in given ray do not undergo any essential changes, i.e. it is expressed by equation (41), which can be written in the shape

$$A_0 = A_{0a}\exp(i\varphi_a), \quad A_{0a} = \left[\left(1 + (z_D - L)^2\right)/a_C^2\right]^{-1/2}, \quad \mathrm{tg}\,\varphi_a = (L - z_D)/a_C. \tag{49}$$

In this situation the light beam electric field in detection plane is expressed by equation

$$u(\vec{r}_D) = E_0 A_{0a} \exp(-k(\psi_{0I} + \psi_{1I}))\exp(ik(\psi_{0R} + \psi_{1R}) + i\varphi_a), \tag{50}$$

and additionally

$$\exp(ik\psi_{1R}) = \exp\left(i\frac{1}{2}k\nu_{Ra}\cos(\beta_R + \beta_c)\right) = \sum_{m=-\infty}^{+\infty} i^m J_m\left(\frac{1}{2}k\nu_{Ra}\right)\exp(im(\beta_R + \beta_c)), \tag{51}$$

where J_m denote the first type Bessel function of order m. In last formula the "classical" series extension was applied, which is used in traditional Raman-Nath diffraction analysis too. Physically this extension denotes, that the light beam interacted with acoustics wave decompose on unlimited quantity of the diffracted beams. From the β_R shape results, that the m order diffracted beam propagate under angle φ_m (relatively to the incident beam direction)

$$\mathrm{tg}\,\varphi_m = \frac{mk_{as}}{kn_0}, \quad k_{as} = k_a\left(1 + \frac{z_D z_s}{a_C^2}\right)\Big/\left(1 + \frac{z_D^2}{a_C^2}\right), \quad z_s = z_I + \Delta z/2. \tag{52}$$

It can be seen, that the diffraction angles depend on the diffraction (z_s) and detection (z_D) places position. In many situations this dependence can be neglected. Diffracted beams have the shifted angular frequency by the value $\Delta\omega_m = m\Omega$ and they have the additional phase shift $\Delta\Phi_m = m(k_a h + \Phi + \beta_c + \pi/2)$. Taking into account, that the acoustic wave frequency is more less then the light wave frequency we can calculate the diffracted beam intensity as

$$I_m = \frac{1}{2}|u_D|^2 \varepsilon_0 \upsilon = \frac{1}{2}E_0^2 \varepsilon_0 \upsilon A_{0a}^2 J_m^2\left(\frac{1}{2}k\nu_{Ra}\right)\exp(-2k(\psi_{0I} + \psi_{1I})) \tag{53}$$

(ε_0 is the free space permittivity and υ is the light wave velocity in the medium). Because the ψ_{1I} depends on time then diffracted beam are intensity modulated. In many cases we can write

$$\exp(-k\psi_{1I}) = \exp(k\nu_{Ia}\sin(\beta_R + \beta_s)) \cong 1 + k\nu_{Ia}\sin(\beta_R + \beta_s), \tag{54}$$

what means, that the modulation occur with acoustic wave frequency. At that time the quantity $k\nu_{Ia}$ denote the modulation depth. If in that conditions the detector measure the mean light intensity (in time) then we obtain

$$\bar{I}_m = \frac{1}{2}E_0^2 \varepsilon_0 \upsilon A_{0a}^2 J_m^2\left(\frac{1}{2}k\nu_{Ra}\right)\exp(-2k\psi_{0I}). \tag{55}$$

When the detector is equipped with the slot parallel to OY axis then expression (55) must be integrated over the y variable. If we assume, that the total power of the incident light beam is equal to P_0 we obtain

$$\bar{I}_{my} = \frac{P_0}{\sqrt{2a^2\pi}\sqrt{1 + (z_D - L)^2/a_C^2}} J_m^2\left(\frac{1}{2}k\nu_{Ra}\right)\exp\left[-\frac{x_D^2}{2a^2\left(1 + (z_D - L)^2/a_C^2\right)}\right], \tag{56}$$

and additionally in most experimental situations

$$\frac{1}{2}k\nu_{Ra} \cong \varsigma \exp[-\alpha(x_R(\tau_s) + h)]\left[\cos^2(\alpha x_I(\tau_s)) + \beta_I^2\right]^{1/2}, \quad \varsigma = k\Delta n \Delta z, \tag{57}$$

(ς denote the Raman-Nath parameter). For $\alpha = 0$, $\beta_I = 0$ and the plane waves ($a \to \infty$) the relation (56) is turned to very well known one in literature[2].

By the expression (56) we can analyse the influence of different experimental parameters on light intensity distribution in diffracted beams. Final results may be illustrated graphically. Exemplary graphs of light intensity distribution in ten following (from 0 to 9) diffraction orders on Fig. 2 is presented. It can be stated, that the diffracted beams are characterised by some "fine" structure. It can be easy proved, that the sum of all these ten intensity distributions practically cover the light intensity distribution in incidence beam.

At the end we must say, that all of the above results can be obtained by wave optics methods. The acoustooptical interaction general analysis of wave beams with arbitrary amplitude and phase distributions was carried out, among other, in works[2, 15]. In work[15] the Bragg type diffraction of the gaussian beam on plane acoustic wave was analysed. In that case the fine structure presence in both diffraction orders was found.

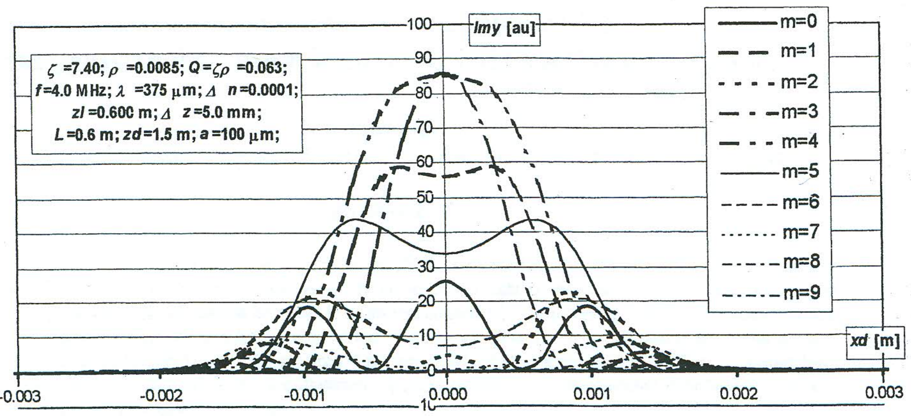

Figure 2. An exemplary light intensity distributions in diffracted beams of order m (from 0 to 9). ζ, ρ and Q denoted the Raman-Nath, Bagavantham-Rao and Klein-Cook parameter respectively.

4. ACKNOWLEDGMENTS

This work was realised in the frame of the KBN (Polish Scientific Committee) grant number 8 T10C 004 12.

5. REFERENCES

1. M. Born and E. Wolf, *Principles of optics*, Pergamon Press, New York, 1965.
2. V. I. Balakshij, V. N. Parygin and L. E. Chirkov, *Physical foundations of acoustooptics*, Radio i Sviaz, Moscow 1985 (in russian).
3. A. Śliwiński, *Ultrasounds and their applications*, WN-T, Warsaw 1993 (in polish).
4. L. Brillouin, „Diffusion de la lumiere et de rayons X par un corps transparent homogene", Annal. De Phys. Ser. 9, **17**, pp. 88-122, 1922.
5. C. V. Raman and N. S. N. Nath, „The diffraction of light by high frequency sound waves", Proc. Ind. Acad. Sci. **2A pt. I**, pp. 406-412, 1935.
6. R. Lucas, „Sur la diffraction de la lumiere par les ondes elastique", Comp. Rend. Acad. Sci. **195**, pp. 1066, 1932.
7. L. Bergman, *Der ultraschall und seine anwendung in wissenschaft und techmik*, Zurich1954.
8. Yu. K. Rebrin, *Optical ray steering in space*, Sov. Radio, Moscow 1977 (in russian).
9. Yu. A. Kravtsov and Ju. I. Orlov, *Geometrical optics in nonhomogeneous media*, WNT, Warsaw 1993 (in polish, translation from russian).
10. Yu. A. Kravtsov, „Complex rays and complex caustics", Radiophysics **10**, pp. 1283-1304, 1967 (in russian).
11. V. A. Baranov and Yu. A. Kravtsov, „Perturbation calculus for rays in nonhomogeneous media", Radiophysics **18**, pp. 52-62 (in russian).
12. A. Yariv and P. Yeh, *Optical waves in crystals*, JW&S, New York (1984).
13. G. A. Deschamps, „Gaussian beam as a bundle of complex rays", Electronics Lett. **7**, 684-685 (1971).
14. B. Keller, W. Streifer, „Complex rays with an application to gaussian beams", JOSA **61**, pp. 40-43 (1971).
15. A. Korpel, *Acousto-optics*, Marcel Dekker Inc., New York 1988.